

Distributed Scheduling in Multiple Access with Bursty Arrivals under a Maximum Delay Constraint

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Abstract

A multiple access system with bursty data arrivals to the terminals is considered. The users are frame-synchronized, with variable sized packets independently arriving in each slot at every transmitter. Each packet needs to be delivered to a common receiver within a certain number of slots specified by a maximum delay constraint. The key assumption is that the terminals know only their own packet arrival process, i.e. the arrivals at the rest of the terminals are unknown to each transmitter, except for their statistics. For this interesting distributed multiple access model, we design novel online communication schemes which transport the arriving data without any outage, while ensuring the delay constraint. In particular, the transmit powers in each slot are chosen in a distributed manner, ensuring at the same time that the joint power vector is sufficient to support the distributed choice of data-rates employed in that slot. The proposed schemes not only are optimal for minimizing the average transmit sum-power, but they also considerably outperform conventional orthogonal multiple access techniques like TDMA.

I. INTRODUCTION

Multiple access channels (MACs) in wireless systems are conventionally studied under a centralized framework, where a base-station/controller regulates the transmission rates and powers of all the users [1]–[5]. This requires global state knowledge of the underlying time-varying processes. The lack of such global knowledge in a MAC leads to decentralized operations. The

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two common time-varying processes in wireless communication are data-arrivals and fading coefficients. Multiaccess under time varying fading models are extensively studied under centralized frameworks [3], decentralized fast-fading setups [5]–[7], or decentralized block-fading models [8], [9]. Notice that the fading MACs above assume an infinite bit-pool model, suitable for mobile applications targeting higher throughputs, without emphasizing the delay requirements. As opposed to these, the current paper focuses on bursty data arrivals to the transmitters, with delay constraints.

Bursty packet arrivals to the terminals are more practical in data networks. A time-slotted fixed fading MAC with frame-synchronized users and independent packet arrivals can effectively model several limited mobility applications, and wireless back-haul services. Packets arrive to the respective queue at each transmitter and needs to be appropriately scheduled through the MAC channel. It is reasonable to assume here that only the respective transmitters and the receiver know the arrival-instants/packet-sizes to each queue [10]. Notice that bursty arrivals pose new challenges, as it may necessitate data scheduling and power control to respect the causality of arrivals as well as delay constraints. While handling arrivals and delays can be challenging in point-to-point channels also, it is even more pronounced in multiuser networks. More specifically, independent arrival processes at the terminals of a MAC will force a distributed operation.

The absence of a centralized controller in a MAC model will lead to random access. However, the name *random access* is traditionally attributed to dynamic network access schemes like ALOHA, CSMA etc. These are extensively studied in literature [11]. In general, the literature related to network access control falls roughly into two categories: (i) *closed loop* control and contention resolution; (ii) *open loop* scheduling and stabilizing queues. ALOHA and CSMA fall into the former group, whereas the latter contains flow control schemes based on buffer and link states [12]. In both models, the objectives typically are to maximize throughput, minimize delay, or both. While the related literature is large, in order to highlight the differences to the model that we consider, let us review some works relevant to our model.

A. Related Literature

Closed loop systems like ALOHA and CSMA typically abstract the physical layer as a bit-pipe, where simultaneous access by several users leads to a collision, or outage [11]. Collision events are sensed or fed back, and are resolved using contention resolution protocols. While sensing the

medium prior to transmission can reduce the chances of collision, appropriate control policies are still needed to adjust the transmission probabilities for achieving optimal throughput [13]. Multi-packet reception capability is also extensively studied, where it is possible to capture information simultaneously from several users, see [14] for some recent advances and references. It is well known that the bit-pipe abstraction of physical layer forms an *unconsummated union* with the information theoretic considerations [15]. Several approaches tried to bridge this gap by studying queuing and scheduling models, by specifying the quality of service constraints by information theoretic quantities like capacity, error exponents etc [16], [17]. Under the assumption of reasonably large blocklengths, these works provide rigorous mathematical foundations on which the utilities like transmission-rate and probability of error can be connected to networking quantities like throughput and delay.

Unlike the statistical multiplexing schemes like ALOHA/CSMA, we consider an information theoretic MAC model with a fixed number of users, each observing an independent arrival process. Thus the variability is not just in the presence or absence of packets, but in the size of the packets itself. Furthermore, the associated delay constraints may necessitate a packet to be broken into sub-packets and transmitted in different slots. In this sense, our model differs from conventional random access. In fact, the model here is more related to cross layer scheduling and control in wireless systems, comprehensively covered in the recent surveys [18], [12], see also the references therein. Notice that bursty packet arrivals to a system can lead to interesting trade-offs between the network layer delay and the transmit-power in physical layer, and intelligent scheduling algorithms are required to achieve optimal performance. Of particular interest are the *open loop* scheduling schemes which choose the transmission parameters such as rate and power based on operating conditions like queue state.

A point to point AWGN link with packet arrivals was considered in [19], with the objective of finding the optimal trade-offs between average power and delay. Optimal schedulers which minimize the average transmit power under an average or max-delay constraint were identified using a dynamic programming (DP) framework. The key observation in [19] is that large savings on transmit power can be obtained by accommodating some more delay within the tolerable limits. This was later extended to other scheduling models [20], and also to networks [21], [22]. Note that all these extensions considered centralized systems where the arrival processes are known to all the terminals. Interestingly, [21] remarks that the ultimate objective of analyzing

centralized schemes is to find good *decentralized* schedulers. We make progress in this direction by presenting optimal decentralized schedulers for a MAC with arrivals, under a maximum delay metric, in the current paper.

In a separate line of work, [23] established the optimal energy-efficient offline scheduling algorithm which meets a single deadline constraint for all the arriving packets over a point-to-point AWGN link. The optimal scheduler in this set up will operate at a low enough transmission-rate, with the rate at any instant being at least as big as the rates employed till that time. This leads to the so-called *move-right* algorithm. An online *lazy* algorithm to vary the transmission rate according to the current backlog was also proposed and shown to have good asymptotic performance in [23], see [24]–[30] for extensions.

Energy-delay tradeoffs for multiuser wireless links with online arrivals were considered in [28], [31]. In particular, [28] considers a wireless downlink with a separate queue for each receiver. The base station has global state-information, and the broadcast nature of the downlink makes it a centralized model. In a more recent work, [31] considers delay aware scheduling in multi-user wireless networks. However, a centralized entity schedules one of the links in each slot. In contrast to [19]–[31], which all had some form of centralized scheduling and control, a decentralized MAC with arrivals is considered in this paper.

Models with both time-variations in arrivals and fading coefficients are also of interest. For example, [24], [25], [32] consider dynamic fading and arrivals for a point-to-point system, whereas [27], [28], [31] analyze centralized multi-user models. In another interesting work, [33] considers a slow-fading distributed MAC, where each user has access only to its own link quality and arrival process, from a collision resolution perspective. Along the same lines, [34] proposes a channel aware ALOHA protocol to exploit multiuser diversity. A centralized scheduler with decentralized power control is considered for contention resolution in [35]. Notice that [33]–[35] do not explicitly address any delay constraints, and employ the underlying physical layer bit-pipe view of random access. Taking a different standpoint, efficient decentralized open-loop schedulers for a fading MAC with arrivals, so as to minimize the average sum-power required to communicate in an outage-free manner, is an interesting problem. To keep the average power bounded, one can assume that the possible fading values of interest are non-zero. This is one of the topics discussed in this paper, for which there seems few prior results.

Perhaps the closest work in literature to the current sequel is the distributed rate-adaptation

framework in a block-fading MAC [9], and its application to energy harvesting [36]. However, both [9] and [36] consider throughput maximization in distributed MACs, and have nothing to do with delay constraints. More specifically, [9] maximizes the throughput under local knowledge of the link fading parameters, whereas [36] achieves the same objective under the distributed knowledge of energy harvesting processes at the transmitters. Interestingly, one of the motivations behind the introduction of a distributed rate-adaptation framework in [8] was the throughput maximization in random access systems. *Broadcasting* is another useful technique to increase the throughput of distributed systems, where depending on the conditions, parts of the data can be correctly decoded [37]. Rate-less coding without any arrivals for distributed multiple access was considered in [38]. As opposed to these, the objective of the current paper is in minimizing average sum-power under a maximum delay constraint. This, in some sense, parallels the problem of throughput maximization in distributed systems [9], [36]. In fact, the approach and techniques here are motivated by [9], [36], this will be evident from the structural similarities of the results presented here.

B. Main Contributions

We consider a L -user AWGN MAC with bursty packet arrivals, as shown in Figure 1. The transmissions are frame-synchronized, and time is divided into slots or blocks (the words ‘slots’ and ‘blocks’ are used interchangeably in this sequel). Assume that variable sized packets independently arrive at the respective terminals at the start of each slot. The packets are to be conveyed to the receiver within D_{max} slots, i.e. a max-delay constraint of D_{max} . Each transmitter, by observing its own data arrival stream, will schedule the transmission rate as well as power in a slot-wise manner such that the arrived data is conveyed before the respective delay constraints. The challenge here is to perform successful data transfer without knowing the exact arrivals at the other terminals, except for the statistics. The word *successful* is used in the sense of transmitted data not being in *outage* for any transmission block. Notice that no arrival in a slot is also allowed, it is considered as a zero sized packet. We consider transmission schemes which will not only guarantee successful communication, but also minimize the average transmit sum-power expenditure. In short, we seek power efficient communication schemes for a distributed MAC with online arrivals.

Notice that we assumed the observation of independent random processes at different transmit-

ters. The techniques here depend crucially on the knowledge at each terminal of the statistics of all time-varying quantities in the system. The MAC receiver is also aware of the realizations of all the random variables in each slot. The statistics are only used in the initial design phase, the proposed communication schemes will still work even if the underlying statistics are perturbed. However, the optimality guarantees do not hold under perturbations. In other words, once the statistics are conveyed, no further information exchange is necessary for designing the distributed communication scheme.

The main contributions of the current paper are:

- 1) An optimal distributed communication scheme for a MAC with independent bursty data arrivals is presented under a unit slot delay constraint on the arriving packets. An explicit power allocation scheme is shown to give an almost closed form solution to the minimal average transmit sum-power (Theorem 8, Section III).
- 2) An optimal distributed power control policy incorporating both time-varying fading and bursty arrivals is presented, for a unit slot delay constraint (Theorem 14, Section IV).
- 3) For a general max-delay constraint of D_{max} , and a fixed fading MAC with independent bursty arrivals, we propose an iterative technique to find optimal schedulers for rate-adaptation and power control (Section V). This effectively addresses the question posed in [21]: “the ultimate goal is to find decentralized schedulers that approach the performance of the centralized scheduler”.

Our results capture the tradeoff between the QoS parameters of delay and required energy/power, for a distributed wireless multiple access model in which several users can simultaneously access the medium. Notice that the users are free to do rate adaptation and power control, while ensuring outage free operations. The trend of tolerable delay being proportional to the achieved energy efficiency is an expected one, this is observed in the distributed MAC model too. However, the results clearly demonstrate that higher energy efficiency and lower delay than conventional schemes can be simultaneously achieved by resorting to the optimal communication schemes presented in this paper.

The techniques here also apply to more general delay constraints than max-delay. However, max-delay is chosen for its simplicity as well as wide application. In particular, the proposed communication schemes can be extended to other delay constraints for which efficient single user schedulers can be identified. Also, the utility of average sum-power is chosen for convenience,

the results can be extended to minimize the weighted average sum-power as well.

The paper is organized as follows. Section II details the system model and notations. Section III considers distributed MACs with fixed fading values and bursty arrivals, under a unit slot delay constraint. In Section IV, we extend the unit slot delay results to the case of dynamically varying fading and bursty arrivals. Then, in Section V, we consider a fixed fading MAC under a general max-delay constraint of D_{max} slots, and propose an iterative algorithm to compute the optimal average sum-power in this case. Simulation results are provided in Section III-C, Section IV-A and Section V-C, to compare the performance of the optimal schemes proposed here with the conventional schemes in literature. Finally, Section VI concludes the paper.

In this paper $\mathbb{E}[X]$ denotes the expectation of random variable X .

II. SYSTEM MODEL

Consider the multiple access system shown in Figure 1, which is referred to as a distributed MAC with bursty arrivals. For L transmitters, the real valued discrete-time model is described by the observed samples

$$Y = \sum_{i=1}^L \sqrt{\alpha_i} X_i + Z,$$

where X_i represents the transmitted symbols from user i . The fading coefficients $\sqrt{\alpha_i}$, $1 \leq i \leq L$ are assumed to be fixed and known to all parties. The noise process Z is normalized additive white Gaussian, independent of all the transmitted symbols. The transmissions take place in a frame-synchronized slotted manner, where each slot (or block) is of length N . The blocklength N is assumed to be large enough for coding and decoding to take place with a sufficiently low error probability.

At the start of each time slot, a variable sized packet arrives independently at each transmitter. We denote the arrival process to terminal i as $A_i[j]$, which implies that $NA_i[j]$ bits arrive at the start of block j to this terminal. The most important aspect of the system that we consider is that each transmitter knows only its own arrival process, i.e. the packet-sizes at rest of the terminals are unknown to each transmitter. However, the statistics of all the arrival processes are available to each party. For simplicity as well as practical relevance, we will assume that $A_i[j]$ are independent and identical across j , each taking values from a finite set \mathcal{A} , with $|\mathcal{A}| < \infty$. Furthermore, we also assume that the arrivals at different terminals are independent, but can be of arbitrary distributions on \mathcal{A} .

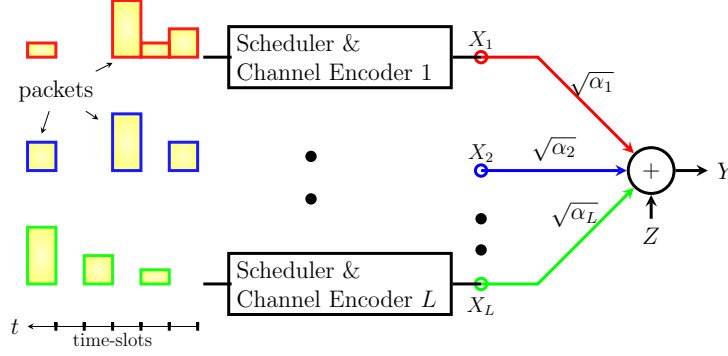


Fig. 1. Distributed MAC with bursty packet arrivals

Assume that each packet is required to be delivered within D_{max} time slots of its arrival. In the system model depicted in Figure 1, each transmitter is shown to have two components, a scheduler and a channel encoder. The scheduler specifies the number of bits to be conveyed in each slot, or equivalently, the transmission rate. Notice that the system allows multi-slot breakup of packets without violating the max-delay of each packet. The channel encoder has to ensure that the scheduled bits in each slot are conveyed correctly to the receiver, i.e. there is no outage. More precisely, we say that the receiver does not encounter outage if the decoding error probability in each block decays exponentially to zero with blocklength, a standard practice in information theory parlance [10], see [17] for a more formal justification. It is well known that any rate-tuple inside the AWGN MAC capacity region will not lead to outage in the above sense. Thus, for a rate-vector (r_1, \dots, r_L) in a block, the channel encoders can ensure successful decoding by choosing Gaussian codebooks with high enough short-term (or per-slot) average transmit power P_i at terminal $i \in \{1, \dots, L\}$ such that

$$\sum_{i \in J} \alpha_i P_i \geq 2^{2(\sum_{i \in J} r_i)} - 1, \forall J \subseteq \{1, \dots, L\}. \quad (1)$$

Thus, for any rate-vector (r_1, \dots, r_L) scheduled in a slot, the transmit powers should obey (1). For a two user MAC model, the set of power-tuples which can support a rate-pair (r_1, r_2) is illustrated in Figure 2 as the shaded portion, which is a contra-polymatroid [3].

Definition 1. A set of power allocation functions $P_i(\cdot), 1 \leq i \leq L$ satisfying (1) for any feasible rate-vector (r_1, \dots, r_L) is called an outage free power allocation.

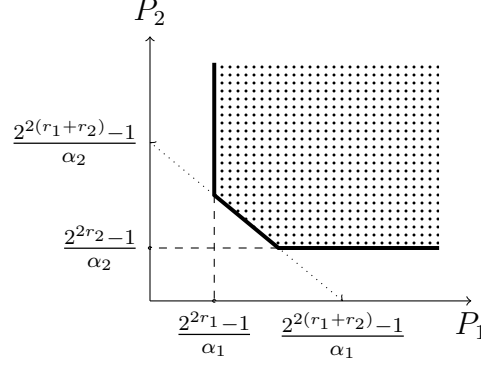


Fig. 2. Set of (P_1, P_2) supporting rate-pair (r_1, r_2)

We consider only outage free power allocations in this paper. In addition, each terminal has to do rate-adaptation, which specifies the number of bits scheduled for transmission in a slot-wise manner, while ensuring the maximal delay constraint. Schemes meeting the max-delay constraint with outage free power allocations are called as *outage free communication schemes*. Since the exact arrivals as well as rate-demands at other terminals are not available, each transmitter makes scheduling decisions based on its own arrival history, along with the statistics of arrival processes at all the terminals. Let $NB_i[j]$ bits are scheduled for slot j by terminal i . In other words, $B_i[j] \in \mathcal{B}_i$ is the transmission rate chosen for slot j at user i . The remaining bits will wait in the queue for future scheduling. At the start of block j , let $N\hat{r}_i[j, d]$ be the number of bits remaining in the i^{th} queue which can afford a delay of at most d more blocks. Note that $\hat{r}_i[j, D_{\max}] = A_i[j]$.

Definition 2. The D_{\max} -dimensional vector $\zeta_i[j] = (\hat{r}_i[j, d], 1 \leq d \leq D_{\max})$ is termed as the state-vector of transmitter i .

At times we may drop the square brackets and call the state-vector as ζ_i . Our objective is to compute the infinite horizon minimum average sum-power expenditure $P_{\text{avg}}^{\min}(D_{\max})$ at the terminals, i.e.

$$P_{\text{avg}}^{\min}(D_{\max}) := \inf_{\Theta} \limsup_{M \rightarrow \infty} \sum_{l=1}^L \frac{1}{M} \mathbb{E} \left(\sum_{j=0}^{M-1} P_l(B_l[j]) \right), \quad (2)$$

where Θ is the set of all *outage free* communication schemes which specify the rate-power

tuples $(B_l[j], P_l(B_l[j]))$, $1 \leq l \leq L, 0 \leq j \leq M-1$, while meeting the maximal delay constraint D_{max} for each packet. The formulation in (2) is actually the infinite horizon average cost minimization problem of a Markov Decision Process (MDP) [39], [40]. Such MDPs already find wide applications in single user scheduling problems [19]. In the MDP formulation, the scheduling actions at terminal l are based on the current value of ζ_l , i.e. the size and delay requirements of the queue backlog. For $1 \leq l \leq L$, let Θ_l^d be the collection of all deterministic outage free strategies $\theta_l : \zeta_l \mapsto (B_l, P_l)$, with $(B_l, P_l) \in \mathcal{B}_l \times \mathbb{R}^+ \cup \{0\}$, such that the packet-delay at user l is at most D_{max} for any $\theta_l \in \Theta_l^d$. Observe that no queue in the system ever builds up, since we have bounded packet-sizes and a maximal delay constraint. Furthermore, in the AWGN MAC setup that we consider, it is also reasonable to assume that the per block average power at a transmitter is continuous in the transmission-rate. These observations allow the following reformulation of (2).

Lemma 3.

$$P_{avg}^{min}(D_{max}) = \sum_{l=1}^L \inf_{\theta_l \in \Theta_l^d} \lim_{M \rightarrow \infty} \frac{1}{M} \mathbb{E} \sum_{j=0}^{M-1} P_l(B_l[j]). \quad (3)$$

Proof: The proof is given in Appendix A. ■

Under the reformulation in Lemma 3, notice that $B_l[j]$ can be taken as the output process of a deterministic scheduler with IID arrivals as inputs. Thus $B_l[j]$ is a stationary ergodic process and we can write [39]

$$P_{avg}^{min}(D_{max}) = \sum_{l=1}^L \inf_{\theta_l} \mathbb{E} (P_l(B_l)),$$

where the random variable B_l has distribution same as the marginal ergodic law of $B_l[j]$. Now we can focus on designing optimal power allocation schemes using the distributions of B_l , $1 \leq l \leq L$. This effectively decouples each transmitter into two components, *viz.* a bit scheduler (BiS) and a channel encoder (CeN). This is illustrated in Figure 3 for a two user MAC model.

Each bit-scheduler (BiS) ensures that the delay constraint D_{max} of every arriving packet is met. In addition to meeting the delay constraint, the BiS works in tandem with the channel encoder (CeN) to improve the overall power efficiency. On the other hand, each CeN operates under a unit delay constraint, ensuring that the bits scheduled by the BiS for every slot are successfully conveyed to the receiver by the end of that slot.

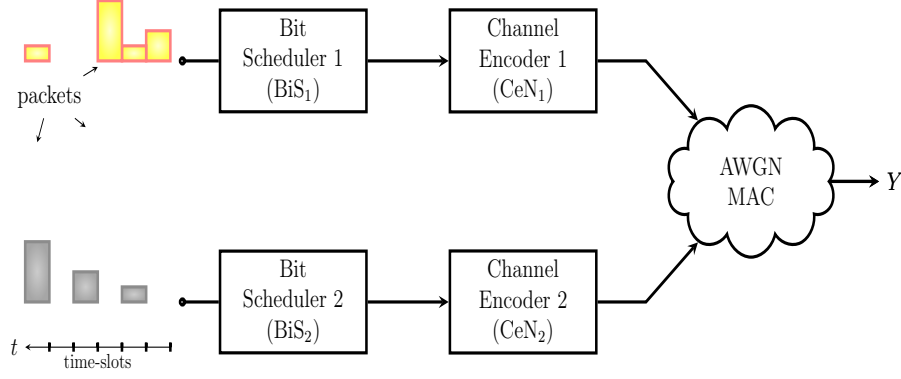


Fig. 3. Decoupling of Transmitters into BiS and CeN

Let the set of L BiSs and CeNs employed at the transmitters be denoted by $\bar{\mathcal{S}}$ and $\bar{\mathcal{P}}$ respectively, we will use S_i to refer to BiS i , and P_k for the power law of CeN k . Thus $S_i : \zeta_i[j] \mapsto B_i[j]$, where $B_i[j] \in \mathcal{B}_i$ is the transmission rate chosen for slot j at user i . When the context is clear, we call $\bar{\mathcal{S}}$ as the scheduling scheme, and $(\bar{\mathcal{S}}, \bar{\mathcal{P}})$ as the communication scheme. The following example illustrates the scheduler actions for a two user MAC.

Example 1. A pair of schedulers with $\mathcal{A} = \{1, 2, 3\}$ and $D_{max} = 2$ is shown in Figure 4, where the row and column indexes respectively indicate the elements of the two dimensional state-vector ζ_i . The matrix entries specify the scheduled transmission-rate for that state-vector. For example, from state $(1, 2)$ at the start of block j for user 1, a transmission-rate of 2 will be chosen. Then, the new state-vector at the start of block $j + 1$ is $(1, A_1[j + 1])$, where the second entry can withstand a delay of 2 units.

$$S_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \end{matrix} \quad S_2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \end{matrix}$$

Fig. 4. Schedulers S_1 and S_2 for $\mathcal{A} = \{1, 2, 3\}$, $D_{max} = 2$

The schedulers shown in Figure 4 output integer-valued transmission rates. However, we can in general allow real-valued rates to be chosen. In practice, the schedulers maybe limited to choose rates which are multiples of some small quanta, or pick one from a given finite set of rates. The effect of quantization on scheduled rates will be illustrated further in the numerical studies of Section V-C.

While the techniques proposed in this sequel extend to any AWGN MAC with independent bursty arrivals, for simplicity, we demonstrate most of the results for a two user MAC. Let the respective fading coefficients be $\sqrt{\alpha_1}$ and $\sqrt{\alpha_2}$, with $\alpha_1 \geq \alpha_2$. In order to proceed with the optimization of (3), we first define a notion of time-sharing between two communication schemes (\bar{S}, \bar{P}) and (\bar{T}, \bar{Q}) . While this notion is useful for our proofs, we reiterate that the optimal distributed schemes in this paper do not employ time division multiple access (TDMA). In fact, the proposed schemes can considerably outperform any variant of TDMA based communication schemes.

A. Time sharing of Scheduling Schemes

The *time sharing* that we introduce here is a bit different from the conventional time division scheme, the latter has different schedulers employed in non-overlapping time intervals. On the other hand, a conceptual time sharing is used here to construct a new scheduler from two existing schedulers, and both schemes will have an impact in each slot of data transfer. In particular, two BiSs are combined to simultaneously operate on the online arrivals as follows.

Definition 4. Consider two scheduling schemes \bar{S} and \bar{T} , both meeting a maximal delay of D_{max} . For $k = S, T$ and $l = 1, 2$, let $B_{k_l}[j]$ denote the rate scheduled in slot j by user l under the scheduling discipline k , when the same arrival process is fed to the two schedulers. For $\lambda \in (0, 1)$, define a new scheduler \bar{S}_λ such that user l schedules a rate $\lambda B_{S_l}[j] + (1 - \lambda) B_{T_l}[j]$ for slot j .

Lemma 5. The scheduler \bar{S}_λ is a valid scheduler meeting the maximal delay constraint of D_{max} .

Proof: Suppose each packet from an arrival process is split into two with a fraction λ of the bits going to the first segment. Let us add dummy bits to each of these segments to make their sizes same as that of the original packet. Thus we obtain two identical streams of data,

and can apply $\bar{\mathcal{S}}$ and $\bar{\mathcal{T}}$ separately on these. Since both $\bar{\mathcal{S}}$ and $\bar{\mathcal{T}}$ meet the delay constraint, we have shown that a fraction λ of the bits get routed through $\bar{\mathcal{S}}$, and the remaining through $\bar{\mathcal{T}}$. Offloading the dummy bits and combining the remaining streams will give us $\bar{\mathcal{S}}_\lambda$. ■

Let us also define a time-sharing on the power-allocation functions. Let $P_{avg}(\bar{\mathcal{S}}, \bar{\mathcal{P}})$ be the average sum-power for the communication scheme $(\bar{\mathcal{S}}, \bar{\mathcal{P}})$.

Definition 6. Consider two power allocations $\bar{\mathcal{P}}$ and $\bar{\mathcal{Q}}$, which allocate powers $(P_1(b_1), P_2(b_2))$ and $(Q_1(b_1), Q_2(b_2))$ respectively to support a rate-pair of (b_1, b_2) . The time-shared power allocation $\bar{\mathcal{P}}_\lambda$ allocates $(\lambda P_1(b_1) + (1 - \lambda)Q_1(b_1), \lambda P_2(b_2) + (1 - \lambda)Q_2(b_2))$ for (b_1, b_2) .

Lemma 7. Consider two communication schemes $(\bar{\mathcal{S}}, \bar{\mathcal{P}})$ and $(\bar{\mathcal{T}}, \bar{\mathcal{Q}})$, and their time-sharing $(\bar{\mathcal{S}}_\lambda, \bar{\mathcal{P}}_\lambda)$. Then, $(\bar{\mathcal{S}}_\lambda, \bar{\mathcal{P}}_\lambda)$ is an outage-free communication scheme and

$$P_{avg}(\bar{\mathcal{S}}_\lambda, \bar{\mathcal{P}}_\lambda) = \lambda P_{avg}(\bar{\mathcal{S}}, \bar{\mathcal{P}}) + (1 - \lambda)P_{avg}(\bar{\mathcal{T}}, \bar{\mathcal{Q}}). \quad (4)$$

Proof: The lemma essentially means that the average sum-power $P_{avg}(\bar{\mathcal{S}}, \bar{\mathcal{P}})$ is convex in the pair $(\bar{\mathcal{S}}, \bar{\mathcal{P}})$. Let us choose any possible scheduled rate-pair (b'_1, b'_2) from $\bar{\mathcal{S}}$. Since $\bar{\mathcal{P}}$ can successfully support this rate-pair, the corresponding received power obeys

$$\alpha_1 P_1(b'_1) + \alpha_2 P_2(b'_2) \geq 2^{2(b'_1 + b'_2)} - 1.$$

Similarly for a rate-pair (b''_1, b''_2) from $\bar{\mathcal{T}}$, we have

$$\alpha_1 Q_1(b''_1) + \alpha_2 Q_2(b''_2) \geq 2^{2(b''_1 + b''_2)} - 1.$$

However,

$$\lambda.(2^{2(b'_1 + b'_2)} - 1) + (1 - \lambda).(2^{2(b''_1 + b''_2)} - 1) \geq 2^{2(\lambda(b'_1 + b'_2) + (1 - \lambda)(b''_1 + b''_2))} - 1, \quad (5)$$

by the convexity of the function 2^x for $x \geq 0$. Thus,

$$\alpha_1 (\lambda P_1(b'_1) + (1 - \lambda)Q_1(b''_1)) + \alpha_2 (\lambda P_2(b'_2) + (1 - \lambda)Q_2(b''_2)) \geq 2^{2(\lambda(b'_1 + b'_2) + (1 - \lambda)(b''_1 + b''_2))} - 1. \quad (6)$$

This guarantees that the scheme $\bar{\mathcal{P}}_\lambda$ can support every rate-pair scheduled by $\bar{\mathcal{S}}_\lambda$. Thus $(\bar{\mathcal{S}}_\lambda, \bar{\mathcal{P}}_\lambda)$ is an outage-free communication scheme. Furthermore, the average sum-power of $(\bar{\mathcal{S}}_\lambda, \bar{\mathcal{P}}_\lambda)$ is same as the λ -linear combination of the average sum-powers individually achieved by $(\bar{\mathcal{S}}, \bar{\mathcal{P}})$ and $(\bar{\mathcal{T}}, \bar{\mathcal{Q}})$ respectively, completing the proof. ■

We now present optimal scheduling schemes for our distributed MAC model. The next two sections discuss the case of unit slot delay constraint, i.e. $D_{max} = 1$.

III. OPTIMAL POWER ADAPTATION UNDER A UNIT DELAY CONSTRAINT

Consider the system shown in Figure 3 with the BiS as an identity function, i.e. all remaining bits are scheduled for transmission at the start of each block, yielding $A_l[j] = B_l[j], \forall j$. This will correspond to a unit slot delay constraint [41]. The arrivals are assumed to be IID over slots, but they have independent, otherwise arbitrary, distributions across users. The IID assumption is for simplicity, the results easily generalize to stationary ergodic processes at the terminals. We will first propose a lower bound to the average sum-power expenditure, and then construct a scheme which meets this bound. The approach here can be visualized as a dual to the MAC throughput maximization framework of [9]. However [9] does not consider arrivals or delay constraints, rather, throughput maximization under a distributed CSIT assumption in time-varying fading models is pursued.

Let the bit-rate random variable B_i at terminal $i \in \{1, 2\}$ be discrete with the marginal law

$$Pr(B_i = b_{ik}) = \lambda_{ik}, 1 \leq k \leq K_i, \quad (7)$$

where the values b_{ik} are assumed to be increasing in k , and K_i is the cardinality of the support of B_i . The CDF of B_i is represented by $\phi_i(b)$. In order to properly combine different integrals, we define an inverse CDF function $b_i(x), i = 1, 2$ for $x \in [0, 1]$, given by

$$b_i(x) = \phi_i^{-1}(x) := \begin{cases} \sup\{b \in \mathbb{R} | \phi_i(b) < x\} & \text{for } 0 < x \leq 1 \\ \sup\{b \in \mathbb{R} | \phi_i(b) \leq x\} & \text{when } x = 0. \end{cases} \quad (8)$$

Using (8), and by a change of variables

$$\mathbb{E}[P_i(B_i)] = \int_{\mathbb{R}^+} P_i(b) d\phi_i(b) = \int_0^1 P_i(b_i(x)) dx. \quad (9)$$

Notice that the integral expression shown in terms of the CDF works even when the underlying distribution is discrete as $b_i(x)$ is defined for all $x \in [0, 1]$. We can now express our result in terms of $b_i(x)$.

Theorem 8. *For a two user MAC with independent bursty arrivals, and respective fading coefficients of $\sqrt{\alpha_1}$ and $\sqrt{\alpha_2}$, $\alpha_1 \geq \alpha_2$, the minimum average sum-power required under a unit slot delay constraint $D_{max} = 1$ is*

$$P_{avg}^{min}(1) = \int_0^{1-\frac{\alpha_2}{\alpha_1}} \frac{2^{2b_2(x)} - 1}{\alpha_2} dx + \int_0^{\frac{\alpha_2}{\alpha_1}} \frac{2^{2(b_2(v+1-\frac{\alpha_2}{\alpha_1})+b_1(\frac{\alpha_1 v}{\alpha_2}))} - 1}{\alpha_2} dv.$$

Proof: Though the expression above appears complex, the minimum sum-power expenditure is simple to evaluate for any set of independent arrival processes. The proof proceeds by starting with the expectation expression in (9) and constructing a suitable lower bound as x traverses from 0 to 1. This is given in the coming subsection. An outage-free communication scheme operating at this average sum-power will then be presented in III-B, thus proving the theorem. ■

A. Lower Bound to $P_{avg}^{min}(1)$

Let us denote $P_i(b_i(x))$ as $\hat{P}_i(x)$, $\frac{\alpha_2}{\alpha_1}$ as α , and take $\bar{\alpha} = (1 - \alpha)$. The expected sum-power can be written as

$$\begin{aligned} \mathbb{E}[P_1(B_1) + P_2(B_2)] &= \int_0^1 P_1(b_1(x)) + P_2(b_2(x)) dx \\ &= \int_0^{\bar{\alpha}} \hat{P}_2(x) dx + \int_{\bar{\alpha}}^1 \hat{P}_2(x) dx + \int_0^1 \hat{P}_1(x) dx \\ &= \int_0^{\bar{\alpha}} \hat{P}_2(x) dx + \int_0^{\alpha} \left(\hat{P}_2(v + 1 - \alpha) + \frac{\hat{P}_1(\frac{v}{\alpha})}{\alpha} \right) dv \end{aligned} \quad (10)$$

$$\geq \int_0^{\bar{\alpha}} \frac{2^{2b_2(x)} - 1}{\alpha_2} dx + \int_0^{\alpha} \frac{\alpha_2 \hat{P}_2(v + 1 - \alpha) + \alpha_1 \hat{P}_1(\frac{v}{\alpha})}{\alpha_2} dv \quad (11)$$

$$\geq \int_0^{\bar{\alpha}} \frac{2^{2b_2(x)} - 1}{\alpha_2} dx + \int_0^{\alpha} \frac{2^{2(b_2(v+\bar{\alpha})+b_1(\frac{v}{\alpha}))} - 1}{\alpha_2} dv. \quad (12)$$

In the above, (10) is obtained by change of variables and combining two integral terms. The inequality (11) results from the fact that an average power of $\alpha_2^{-1} (2^{2b} - 1)$ is required to transmit at a rate of b bits per transmission by user 2, even when the other user is absent. Furthermore, to support the rate-pair (b_1, b_2) , we know from (1) that

$$\alpha_1 P_1 + \alpha_2 P_2 \geq 2^{2(b_1+b_2)} - 1, \quad (13)$$

which will in turn justify (12). Thus our converse proof is complete.

B. Scheme achieving $P_{avg}^{min}(1)$

We will specify an iterative outage free communication scheme with an average power of $P_{avg}^{min}(1)$ given in Theorem 8. Notice that it is sufficient to specify the corresponding transmit

power against the rates given by $b_i(x)$, $0 \leq x \leq 1$, these are the inverse CDF values defined in (8).

Let us denote $\frac{\alpha_2}{\alpha_1} = \alpha$, and $\bar{\alpha} = 1 - \alpha$. Motivated by (12), we can assign

$$P_2(b_2(x)) = \frac{2^{2b_2(x)} - 1}{\alpha_2}, \quad 0 \leq x \leq 1 - \alpha, \quad (14)$$

to match the first term there. The rest of the allocations are chosen to match the remaining terms in (12). To this end, define

$$m = \max\{k : \sum_{i=1}^{k-1} \lambda_{2i} < \bar{\alpha}\},$$

where λ_{2i} is given in (7). Now, consider the set

$$\Gamma_{\text{no}} := \{0\} \bigcup \left\{ \sum_{i=1}^j \lambda_{2i} - \bar{\alpha}, m \leq j \leq K_2 \right\} \bigcup \left\{ \sum_{i=1}^j \alpha \lambda_{1i}, 1 \leq j \leq K_1 \right\}. \quad (15)$$

Let us arrange the elements of Γ_{no} in ascending order to obtain an ordered set Γ . Observe that the set $\Gamma := \{\gamma_0, \gamma_1, \dots, \gamma_{|\Gamma|-1}\}$ includes all the CDF values of B_1 scaled by a factor α , in addition to other terms. Thus the set $\{b_1(\frac{\gamma_k}{\alpha}), \forall k\} = \{b_{1k}, \forall k\}$, where b_{1k} is the k^{th} biggest bit-rate required at user 1. Similarly $\{b_2(\gamma_k + \bar{\alpha}), \forall k\} = \{b_{2k}, k \geq m\}$. The power allocations are iteratively specified for the corresponding values in the increasing order of γ_i . After each assignment, the iterative procedure computes the power for a hitherto unassigned bit-rate value, chosen based on the ordered list Γ . By convention, user 2 is updated before the other whenever possible. Using the short notation,

$$P_{u,v}^s := 2^{2(b_2(u+1-\frac{\alpha_2}{\alpha_1})+b_1(\frac{v\alpha_1}{\alpha_2}))} - 1,$$

we are all set to specify the power allocations.

Definition 9. Let $P_1(\cdot)$ and $P_2(\cdot)$ be two power allocation functions such that

$$P_2(b_2(x)) = \frac{2^{2b_2(x)} - 1}{\alpha_2}, \quad 0 \leq x \leq 1 - \frac{\alpha_2}{\alpha_1} \quad (16)$$

and for $\gamma_i \in \Gamma$, $0 \leq i \leq |\Gamma| - 1$,

$$\alpha_1 P_1\left(b_1\left(\frac{\gamma_i \alpha_1}{\alpha_2}\right)\right) = P_{\gamma_i, \gamma_i}^s - \alpha_2 P_2\left(b_2\left(\gamma_i + 1 - \frac{\alpha_2}{\alpha_1}\right)\right) \quad (17)$$

$$\alpha_2 P_2\left(b_2\left(\gamma_{i+1} + 1 - \frac{\alpha_2}{\alpha_1}\right)\right) = P_{\gamma_{i+1}, \gamma_i}^s - \alpha_1 P_1\left(b_1\left(\frac{\gamma_i \alpha_1}{\alpha_2}\right)\right). \quad (18)$$

Recall that Γ is the set given in (15) arranged in the ascending order.

Lemma 10. *The power allocations given in (16) – (18) achieve $P_{avg}^{min}(1)$ over a two user distributed MAC with bursty arrivals.*

Proof: It is clear that the transmit powers can be chosen as mentioned in the lemma. On close observation of our achievable scheme, we have matched the terms given in the derivation of the lower bound in Section III-A with equality. This will guarantee that our scheme indeed has the minimum possible average power expenditure over a distributed MAC with bursty arrivals and a unit delay constraint. The only missing part is to show that every transmission rate-pair corresponding to the incoming packets can be sustained without outage by the chosen power allocation. This is proved in the next section for the more general case of bursty arrivals as well as dynamic fading, see Lemma 15. The proof of Lemma 10 is now complete. ■

Remark 11. *The proof of Lemma 10 can be adapted to continuous-valued distributions on the arrivals $A_i[j], i = 1, 2$, and also to arbitrary stationary ergodic arrival processes which are independent across the terminals. The former case is detailed in Appendix F.*

C. Simulation Study

Let us now study a simple example to show the utility of the proposed results. Consider a two user MAC system with fading coefficients 1 and $\sqrt{\alpha}$ respectively. Let the required bit-rate in a slot be chosen from $\{1, 2\}$ and the arrival law at each terminal be based on independent and identical Bernoulli random variables with $Pr(B_i = 1) = 0.75, i = 1, 2$. Let us first compare the sum-power of our scheme with two TDM-based schemes. In simple TDM (S-TDM), users share each slot equally among them, whereas in generalized TDM (G-TDM), the fraction of time allotted to a user is optimized to minimize the total transmit power.

Figure 5 compares the power expenditure when the link parameter α is varied in $[0.2, 1]$. The average sum-power for the optimal decentralized scheme is shown as ‘Decentral’. When α moves away from 1, it is evident that there is considerable advantage in using the proposed optimal scheme, over alternatives like TDMA. For a lower bound, we have also plotted the average sum-power of an optimal centralized scheme (Centralized), where each terminal has the global knowledge of arrivals at all the users.

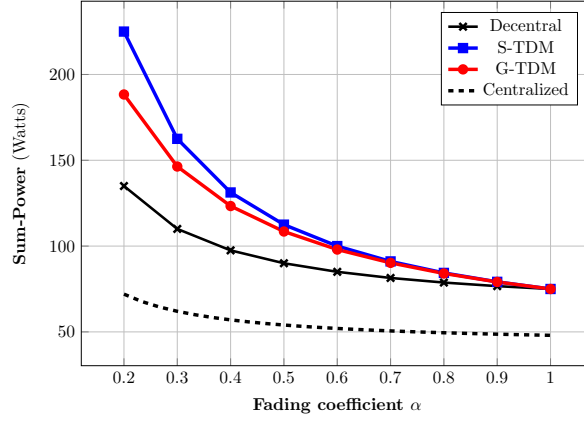


Fig. 5. Optimal Vs TDM for two user distributed MAC with $\alpha_1 = 1, \alpha_2 = \alpha, D_{max} = 1$.

D. Structural Properties of Decentralized Power Allocation

Before generalizing the optimal decentralized schemes, let us highlight some procedural and structural aspects of the optimal decentralized power allocation, the latter are used in the coming sections.

Observe that each terminal has access to the causal knowledge of its own arrival process, along with the statistics at all the terminals. Before the start of any communication, each user can compute its power allocation as a function of the rate requirement. This is done using Definition 9, which only relies on the global statistics. For communicating, the pre-computed power allocation is used to map each arrived rate in a slot to a corresponding transmit power, and a corresponding codeword. This only requires individual knowledge of the arrivals at each terminal. Remarkably, the distributed choice of powers never leads to outage in any block. In other words, the chosen power tuple can sustain the arrived rate vector requirement, as the resulting MAC capacity region is guaranteed to contain the operating rate-pair. In addition, the scheme also minimizes the average sum-power consumption, thus making it optimal.

Let us now list some structural aspects.

Lemma 12. *Each of the power allocation functions $P_i(\cdot), i = 1, 2$ given in Definition 9 is convex in the rate.*

Proof: The proof is given in Appendix D. ■

Notice further that though the power-allocations in Lemma 10 are given for a set of rates specified by (8), the iterations can be continued to extrapolate for higher rate-values, if desired. This can be done by adding suitable dummy rates of zero probability. In addition, one can also extend each allocation to any continuous interval of rates by time-sharing. Lemma 7 guarantees that the resulting communication scheme is outage free. We summarize these observations as a remark.

Remark 13. *Using the power allocation scheme in Lemma 10, we can define a single user scheduler with rate-power characteristics $P_l(b), b \in [0, |\mathcal{B}_l|]$ at terminal l , using time-sharing and extrapolation.*

See Figure 11 for an illustration of the rate-power characteristics. Let us now incorporate dynamic fading to our model.

IV. DYNAMIC CHANNELS AND BURSTY ARRIVALS

Consider a scalar two user discrete-time AWGN MAC with independent bursty arrivals, where the channel coefficients also vary independently across links. Each user knows its own transmission-rate requirement as well as its fading coefficient at the start of the block. Let the arrivals to terminal i be IID with the required rate distribution $Pr(B_i = b_{ik}) = p_{ik}$. The channel H_i undergoes independent block fading with $Pr(H_i = h_{ik}) = q_{ik}$. We assume a finite number of positive fading values for each link in our MAC model. Let us arrange b_{ik} and h_{ik} such that they are increasing in k for each i . For $i = 1, 2$, let ϕ_i be the CDF of the arrival process B_i , and ψ_i be the CDF of H_i . The objective is to find the power allocation schemes $P_i(b_{ij}, h_{ik}), i = 1, 2$ which minimize the average sum-power, i.e.

$$P_{avg}^{min}(1) = \min_{P_1(\cdot, \cdot), P_2(\cdot, \cdot)} \mathbb{E}_{\phi_1, \psi_1} (P_1(B_1, H_1)) + \mathbb{E}_{\phi_2, \psi_2} (P_2(B_2, H_2)). \quad (19)$$

Recall that $P_i(\cdot, \cdot)$ only depends on (B_i, H_i) due to the distributed system assumptions. Let $|\mathcal{B}_i|$ and $|\mathcal{H}_i|$ denote the cardinality of the sample space of B_i and H_i respectively. Define $\alpha_{0|\mathcal{H}_1|} = 0$ and $\beta_{0|\mathcal{H}_2|} = 0$, and let

$$\alpha_{jk} = \alpha_{(j-1)|\mathcal{H}_1|} + \sum_{l=1}^k \frac{p_{1j}q_{1l}}{h_{1l}^2}, \quad 1 \leq j \leq |\mathcal{B}_1|, \quad 1 \leq k \leq |\mathcal{H}_1|$$

$$\beta_{jk} = \beta_{(j-1)|\mathcal{H}_2|} + \sum_{l=1}^k \frac{p_{2j}q_{2l}}{h_{2l}^2}, \quad 1 \leq j \leq |\mathcal{B}_2|, \quad 1 \leq k \leq |\mathcal{H}_2|. \quad (20)$$

Let us illustrate these definitions and notations by an example.

Example 2. Take $B_1 \in \{2, 3\}$, $H_1 \in \{1, \sqrt{3}\}$, $B_2 \in \{1, 2\}$, $H_2 \in \{1, \sqrt{2}\}$, with $Pr(B_1 = 2) = \frac{1}{3}$, $Pr(H_1 = 1) = \frac{1}{4}$, $Pr(B_2 = 1) = \frac{1}{4}$, $Pr(H_2 = 1) = \frac{1}{2}$. The state-pairs (b, h) for each distribution



Fig. 6. Joint CDFs of arrivals and fading

can be lexicographically ordered, see the directed paths shown in Figure 6. Using (20), we can identify

$$(\alpha_{02}, \alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}) = \left(0, \frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}\right) \quad \text{and} \quad (\beta_{02}, \beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}) = \left(0, \frac{1}{8}, \frac{3}{16}, \frac{9}{16}, \frac{3}{4}\right).$$

These values are marked in Figure 7, where a dummy value $d_0 = \beta_{22} - \alpha_{22}$ was added at the base of the first vector to equalize the heights.

Observe that the cumulative values (labeled as β_{ij} and $d_0 + \alpha_{ij}$) shown in Figure 7 do not correspond to actual CDFs, we call them a pseudo CDF-pair. Notice the dashed levels marked by horizontal lines, these values play an important role in our iterative power allocation. The key idea which we take forward from this example is to allocate power iteratively to each pair connected by a horizontal dashed level.

Let us generalize this example, and lexicographically enumerate the tuples (B_i, H_i) to construct a pseudo-CDF pair as in Figure 7. Without loss of generality, assume $\beta_{|\mathcal{B}_2||\mathcal{H}_2|} \geq \alpha_{|\mathcal{B}_1||\mathcal{H}_1|}$. Using (20), define two maps χ_1 and χ_2 as follows.

$$\begin{aligned} \chi_1(0, 0) &= \beta_{|\mathcal{B}_2||\mathcal{H}_2|} - \alpha_{|\mathcal{B}_1||\mathcal{H}_1|} \\ \chi_1(B_1 = b_{1j}, H_1 = h_{1k}) &= \chi_1(0, 0) + \alpha_{jk}, \quad 1 \leq j \leq |\mathcal{B}_1|, \quad 1 \leq k \leq |\mathcal{H}_1| \end{aligned}$$

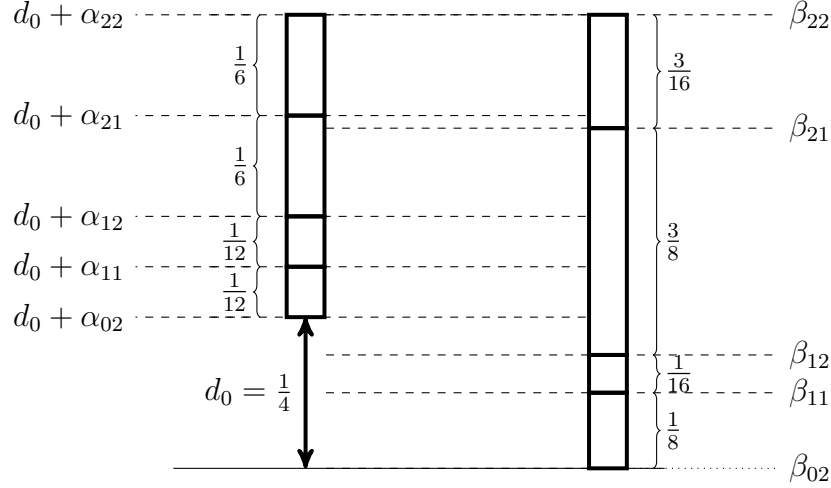


Fig. 7. Pseudo CDF-pair

$$\chi_2(B_2 = b_{2j}, H_2 = h_{2k}) = \beta_{jk}, \quad 1 \leq j \leq |\mathcal{B}_2|, \quad 1 \leq k \leq |\mathcal{H}_2|.$$

Let $\text{Range}(\chi_i)$ denote the range of the map χ_i , and take $\Gamma := \text{Range}(\chi_1) \cup \text{Range}(\chi_2)$, with the elements indexed in the ascending order. To clarify, in Figure 7, the set $\Gamma := \{\gamma_0, \dots, \gamma_{|\Gamma|-1}\}$ is simply the ordered collection of the dashed horizontal levels shown there. Let us also define the inverse map of $\chi_i, i = 1, 2$ by

$$(b_i(\gamma_l), h_i(\gamma_l)) = \max\{(b_{ij}, h_{ik}) : \chi_i(b_{ij}, h_{ik}) \leq \gamma_l\}, \quad (21)$$

where $\gamma_l \in \Gamma$, and the maximum is in the lexicographical order. We now present an optimal power allocation scheme. Like in Section III, the iterative scheme proceeds in the increasing order of γ_l , and power will be allocated at each step to the inverse of $\gamma_l \in \Gamma$, for a hitherto unallocated pair of rate and fading-value at a user.

In the following theorem, $P_i(b_i(\gamma_l), h_i(\gamma_l))$ is denoted as $P_i(l)$ for brevity. Denote the smallest index in $\{0, \dots, |\Gamma| - 1\}$ such that γ_l corresponds to a positive rate for at least one of the users as l^* . Clearly $P_i(l) = 0$ if $l < l^*$, as there is no need for any allocation.

Theorem 14. *The power allocation functions $P_1(\cdot)$ and $P_2(\cdot)$ given by*

$$h_1^2(\gamma_{l-1})P_1(l-1) + h_2^2(\gamma_l)P_2(l) = 2^{2(b_1(\gamma_{l-1})+b_2(\gamma_l))} - 1 \quad (22)$$

$$h_1^2(\gamma_{l-1})P_1(l-1) + h_2^2(\gamma_{l-1})P_2(l-1) = 2^{2(b_1(\gamma_{l-1})+b_2(\gamma_{l-1}))} - 1 \quad (23)$$

for $l^* < l \leq |\Gamma| - 1$, with the initial power allocation satisfying

$$h_1^2(\gamma_{l^*})P_1(l^*) + h_2^2(\gamma_{l^*})P_2(l^*) = 2^{2(b_1(\gamma_{l^*})+b_2(\gamma_{l^*}))} - 1 \quad (24)$$

$$h_i^2(\gamma_{l^*})P_i \geq 2^{2b_i(\gamma_{l^*})} - 1, \quad i = 1, 2, \quad (25)$$

achieve

$$\mathbb{E}P_1(B_1, H_1) + \mathbb{E}P_2(B_2, H_2) = P_{avg}^{min}(1). \quad (26)$$

Proof: The proof can be found in Appendix B. ■

It now remains to be shown that the power allocation scheme in Theorem 14 is outage free.

Lemma 15. *The power allocation given in (22) – (25) is an outage free scheme over a distributed MAC with bursty arrivals.*

Proof: The proof is given in Appendix C. ■

We have thus shown an optimal scheme which achieves $P_{avg}^{min}(1)$, and is outage free. Before embarking on a simulation study, some comments are in order. It should be noted that the channel values are not ordered monotonically while constructing the pseudo-CDF pair (see Figures 6-7), it is enough to take the required transmission rates at each user in the increasing order while the powers are iteratively assigned. In particular, the fading values and their probabilities play a role in the construction of the pseudo-CDF pair.

Remark 16. *Suppose that after evaluating the pseudo-CDF pair, we replace every fading value by unity. The power allocation in Theorem 14 will now specify the required received power for each transmission-rate chosen by a user. Clearly, the transmit powers at the CeNs of the original MAC can be found by appropriate scalings.*

Notice that for each $\gamma_l \in \Gamma$, (21) defines a pair of values at user $i \in \{1 \leq i \leq L\}$, let I_l denote the ordered collection of these L pairs.

Remark 17. *The knowledge of the set $\{I_l, 0 \leq l \leq |\Gamma| - 1\}$ at each user is sufficient to specify the complete power-allocation scheme. Thus, even the knowledge of the statistics is redundant while designing the communication scheme, once the users have access to $\{I_l, 0 \leq l \leq |\Gamma| - 1\}$.*

An astute reader might have observed that our approach in this section differed slightly from

the exposition in Section III. While the marginal CDFs of the arrivals were used in the power allocations of Section III, we employed pseudo-CDFs here. The latter approach saved us from an explosion of notations in the presence of dynamic fading. We now detail how Lemma 15 will imply Lemma 10, this will also explain the equivalence of the two approaches.

Suppose we have a fixed fading MAC with the respective fading power gains α_1 and α_2 with $\alpha_1 \geq \alpha_2$. Let ϕ_1 and ϕ_2 denote the respective marginal CDFs of the rate arrivals. If $P_a^m(\alpha_1, \alpha_2)$ denotes the minimum average sum-power for this MAC with bursty arrivals under a unit slot delay constraint, then

$$P_a^m(\alpha_1, \alpha_2) = \alpha_2 P_a^m\left(\frac{\alpha_1}{\alpha_2}, 1\right).$$

Thus, we can equivalently find the minimal average power for a two user MAC with fading power gains $(\frac{\alpha_1}{\alpha_2}, 1)$. For the latter channel, suppose we arrange the values of ϕ_1 and ϕ_2 as in Figure 7, and generate the ordered set Γ . Clearly, $d_0 = 1 - \frac{\alpha_2}{\alpha_1}$ and $\beta_{|\mathcal{B}_2||\mathcal{H}_2|} = 1$, i.e. the height of the graph is unity. Furthermore, using (21)

$$\{(b_1(\gamma_l), b_2(\gamma_l)), 0 \leq l \leq |\Gamma| - 1\} = \left\{ \left(\phi_1^{-1}\left(\frac{\alpha_1(x - d_0)}{\alpha_2}\right), \phi_2^{-1}(x) \right), 0 \leq x \leq 1 \right\}.$$

Observe that the RHS is exactly the set of rate-pairs for which Lemma 10 allocated the minimum required transmit power. Thus, for each $0 \leq l \leq |\Gamma| - 1$, the power allocation for the pair $(b_1(\gamma_l), b_2(\gamma_l))$ is identical in Lemma 10 as well as Lemma 15. Therefore, the allocation in Lemma 10 is a special case of Lemma 15.

A. Simulation Study

Let us now compare the performance of the proposed schemes with TDMA as well as centralized schemes. A generalized TDMA scheme (G-TDM) is used in the simulations below for comparisons, where the fraction of the time given to a user is optimized to get the maximum time-shared sum-rate. The optimal centralized scheme is as follows.

Centralized Scheme: In a centralized scheme, each user knows the global CSI as well as the rate-requirements at all terminals. While (1) still needs to be satisfied for each rate-vector, one can achieve equality in that equation, thus reducing the required average transmit-power in comparison with a decentralized system. With the channel coefficients $(\sqrt{\alpha_1}, \sqrt{\alpha_2})$, minimum transmit sum-power to support the rate-tuple (b_1, b_2) in a slot can be evaluated as

$$\min P_1 + P_2 \text{ subject to: } \sum_{i \in J} \alpha_i P_i \geq 2^{2(\sum_{i \in J} b_i)} - 1, \forall J \subseteq \{1, 2\}. \quad (27)$$

The feasible power-pairs which can support the rate-pair (b_1, b_2) is a contra-pentagon, similar to that shown in Figure 2. Clearly (27) can be solved by operating at the corner-points of the contra-pentagon. In particular, the optimal operating point is always chosen from the line $\alpha_1 P_1 + \alpha_2 P_2 = 2^{2(b_1+b_2)} - 1$. If $\alpha_1 < \alpha_2$ we can take $\alpha_1 P_1 = 2^{2b_1} - 1$, otherwise we take $\alpha_2 P_2 = 2^{2b_2} - 1$. Notice that if $\alpha_1 = \alpha_2$, one can operate anywhere on the dominant face.

In the first simulation below, the effect of variations in the fading statistics on the total power consumption is studied. Let H_2 be uniformly distributed in $\{1, \dots, 5\}$, and H_1 be uniformly distributed in $\{\gamma_a, 2\gamma_a, 3\gamma_a, 4\gamma_a, 5\gamma_a\}$, where γ_a is a positive parameter capturing the asymmetry in the links for the two users. Assume that the arrivals for user $i \in \{1, 2\}$ are chosen with probability

$$Pr(B_i = k - 1) = \frac{p_i(1 - p_i)^{(k-1)}}{[1 - (1 - p_i)^5]}, \quad 1 \leq k \leq 5. \quad (28)$$

Notice that $B_i + 1$ is a truncated Geometric distribution. The parameter p_2 is taken to be 0.25 for all the numerical computations below.

Figure 8 compares the average sum-power expenditure when the link asymmetry parameter γ_a is varied from 1 to 100, while keeping $p_1 = p_2 = 0.25$. Clearly, when the statistical laws are identical at both the users, the decentralized system and G-TDM give similar performance, whereas there is a lot to be gained by centralized operations. However, as the fading laws become more asymmetric, the optimal decentralized schemes perform superior to G-TDM.

Let us now study the effect of variability in arrival distributions as well. Let H_2 be uniform in $\{1, 2, 3, 4, 5\}$, and H_1 be independently and uniformly taken from $\{\gamma_a, 2\gamma_a, \dots, 5\gamma_a\}$. Let us fix the parameter p_2 in (28) at 0.25, and vary p_1 in an appropriate range.

Figures 9 and 10 plot the average sum-power as a function of the ratio p_1/p_2 for $\gamma_a = 1$ and $\gamma_a = 10$ respectively. Note that for $\gamma_a = 1$ and $p_1 = p_2$, the two users are statistically identical and hence the decentralized scheme has performance similar to G-TDM. As the ratio p_1/p_2 increases, the probability of lower sized packets at user 1 increases, hence the required average sum-power diminishes for all the schemes. However, it is evident that the proposed scheme outperforms G-TDM. Similarly, for $\gamma_a = 10$, the decentralized scheme is almost identical to TDMA when $p_1/p_2 \approx 2.8$, but has superior performance in other ranges.

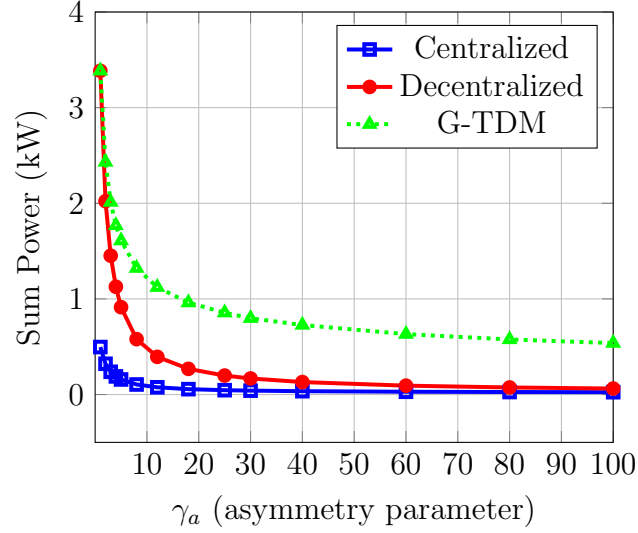


Fig. 8. Decentralized schemes vs TDMA and centralized schemes, $D_{max} = 1$

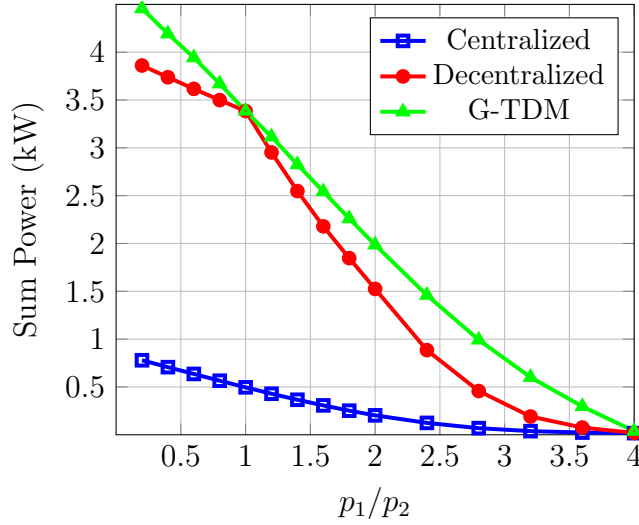


Fig. 9. Sum power versus p_1/p_2 , with the probability parameter $p_2 = 0.25$, asymmetry parameter $\gamma_a = 1$.

V. DISTRIBUTED SCHEDULING UNDER A GENERAL MAX-DELAY CONSTRAINT

So far we have considered a distributed MAC with bursty arrivals under a unit slot delay constraint. A unit-slot delay is a very stringent requirement, relaxed QoS guarantees are more applicable. Let us now consider the widely employed max-delay constraint, i.e. each packet

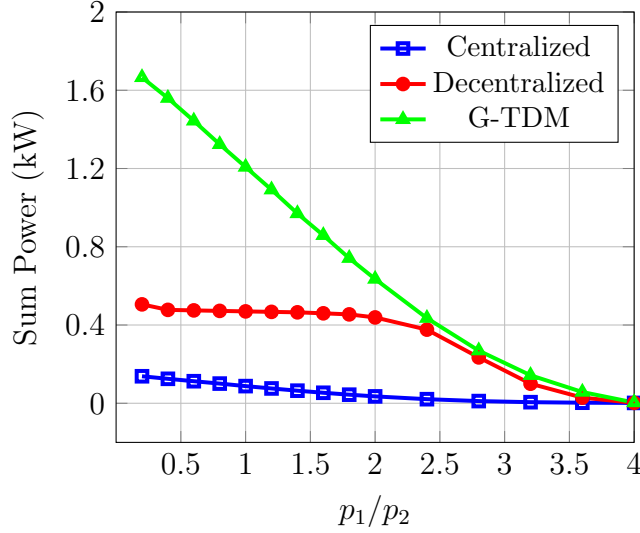


Fig. 10. Sum power versus p_1/p_2 , with the probability parameter $p_2 = 0.25$, asymmetry parameter $\gamma_a = 10$.

should be delivered before D_{max} slots, where $D_{max} \geq 1$ is some specified integer [20]. While we can also allow a separate max-delay constraint for each queue, this will only add notational burden. Since our primary motivation is to analyze the relaxation of delay requirements, we will consider a MAC with fixed fading coefficients and bursty arrivals in this section.

It was already shown in Section II that the operations of BiS and CeN can be decoupled at each transmitter (see Figure 3). More specifically, the CeN $P_i, i = 1, 2$ operates under a unit delay constraint on the scheduled bits from its corresponding BiS S_i . Furthermore, each CeN encounters a stationary ergodic arrival process, as opposed to the IID inputs considered in the previous section. As observed in Remark 11, this can be readily handled by the power allocations in Lemma 10, by using the stationary marginal CDFs there. Furthermore, Remark 13 enables us to construct a suitable rate-power characteristics $P_l(b), 0 \leq b \leq |\mathcal{B}_l|$ for user $l \in \{1, 2\}$.

Figure 11 illustrates the rate-power curve for one of the schedulers specified in Figure 4, where we have taken $\alpha_1 = 10, \alpha_2 = 1$ and uniform arrivals in $\{1, 2, 3\}$. The power allocation $P_1(b_1)$ for rates $b_1 \in [0, 4]$ is shown, where $\{B_1 = 4\}$ is an additional dummy state.

The following local relationship is immediate in lieu of Remark 13.

Claim 18. *For an optimal outage free communication scheme $(\bar{\mathcal{S}}, \bar{\mathcal{P}})$ at the transmitters, the*

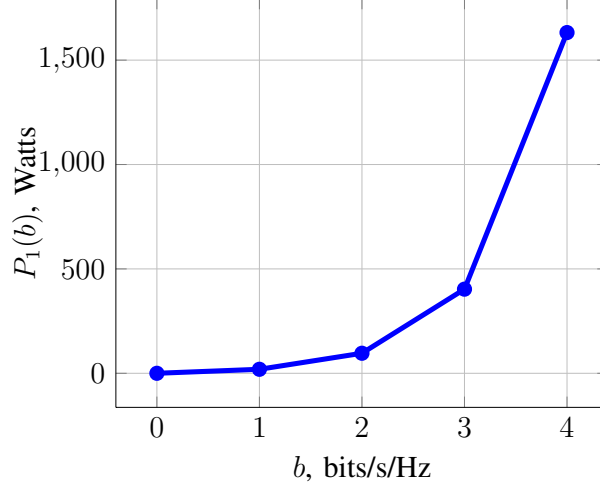


Fig. 11. Rate-Power Characteristic at CeN P_1

scheduler S_i at BiS i is an optimal single-user scheduler for the power allocation function $P_i(\cdot)$.

Proof: Assume on the contrary that some (S_i, P_i) does not meet the asserted property. By keeping all other schedulers and power allocations the same, we can decrease the average sum-power by choosing an optimal S_i for the given P_i . ■

Let $S_{\text{su},i}(P_i)$ denote the optimal single user stationary scheduling policy when the rate-power characteristic at terminal i is given by the function $P_i(\cdot)$. Optimal single user scheduling is a reasonably well understood topic [19], [20], typically solved by dynamic programming, see [39] for a detailed exposition and relevant examples. Using the optimal single schedulers $S_{\text{su},i}(\cdot)$, we now present an iterative algorithm to evaluate the optimal average sum-power required to successfully transport the arriving data in a distributed fashion.

A. Optimal Scheduling Algorithm

Algorithm IterOpt

- 1: The initial power policy \bar{P} is taken as the optimal unit slot delay allocation.
- 2: For \bar{P} , find the optimal single user stationary schedulers $S_{\text{su},i}(P_i)$, $i = 1, 2$.
- 3: Perform optimal unit slot delay power allocation for the new set of marginal rate distributions at the BiSs.
- 4: Go back to Step 2 using the power allocations from the last step.

Algorithm IterOpt is terminated when the required average sum-power becomes invariant. Notice that we are performing an alternate minimization or Gauss-Siedel minimization on a convex (not strictly) utility [42]. Interestingly, in spite of not having strict convexity, the algorithm is guaranteed to converge to the optimal value, when optimized over the set of schedulers $\bar{\mathcal{S}}$ meeting the maximal delay constraint. Let P_{HALT}^* be the terminal average sum-power given by Algorithm IterOpt.

Proposition 19. *Algorithm IterOpt terminates by achieving the optimal average sum-power; i.e. we have $P_{HALT}^* = P_{avg}^{min}(D_{max})$.*

Proof: The proof is given in Appendix E. ■

Step 2 of the algorithm required the availability of optimal single user schedulers $S_{su,i}(\cdot)$ for each of the given convex rate-power characteristics. This involves solving a DP similar to [19] at each BiS, where computational approaches seem necessary.

B. Single User Scheduling

Recall that for a given power function $P_i(\cdot)$ and buffer state $\zeta_i[j]$ (see Definition 2), the BiS S_i decides an optimal action by choosing an appropriate transmission rate r for slot j . In Algorithm IterOpt, we indeed assumed the availability of an optimal single user scheduler. The optimal scheduling policy is identified typically by dynamic programming approaches [39]. While closed form solutions are not always available, a computational approach known as value iteration algorithm (VIA) can numerically determine the optimal schedules, by solving the Bellman equation for the corresponding discounted cost problem given by

$$V_{j+1}(s) = \min_a \{P(a) + \sum_{s'} \gamma Pr(s'|s, a) V_j(s')\}. \quad (29)$$

Here, j denotes the iteration number, s is the D_{max} dimensional vector of the current buffer state, and $P(a)$ is the power required for the action (transmission-rate) a . The function $Pr(s'|s, a)$ is the probability of buffer going from state s to state s' under the action a , and γ is a discount factor, taken slightly below unity.

In the VIA, the scheduled rate a can take any value from $[s_1, \sum_{i=1}^{D_{max}} s_i]$, in steps of Δ , which is the step-size parameter. The step-size can be chosen appropriately to improve either the speed or accuracy. In particular, integer-valued schedulers can be obtained by setting $\Delta = 1$. Note that

the objective function is non-decreasing with $\Delta \in (0, 1]$. Since $P(a)$ is convex in action a (see Lemma 12), the VIA will converge for each Δ , specifying the optimal scheduler for the power allocation function at each user. We now illustrate Algorithm IterOpt by an example.

Example 3. Let us take $D_{max} = 2$, $\alpha_1 = 10$, $\alpha_2 = 1$, and assume both the arrivals to be uniform in $\mathcal{A} = \{1, 2, 3\}$. We can start with the initial schedulers as shown in Figure 4, which are designed using a TDMA based power-allocation. Using a step-size of $\Delta = 1$ (integer-valued schedulers), Algorithm IterOpt outputs the schedulers S_1^{final} and S_2^{final} shown in Figure 12, after two iterations.

$$S_1^{final} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} \end{matrix} \quad S_2^{final} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \end{matrix}$$

Fig. 12. Schedulers S_1 and S_2 after iterations

C. Simulation Study

We now demonstrate the advantages of using the proposed iterative power minimization framework over conventional TDMA-based schemes, or the robust scheduling framework of [20]. The available slot is equally shared between the users in the TDMA scheme employed for comparisons here. The examples below are taken to be simple enough, yet they capture the intrinsic operational details, and expected performance enhancements. Let us consider a two user MAC system with fixed channel values of 1 and $\sqrt{\alpha}$ respectively. We take arrivals to be uniform in $\mathcal{A} = \{1, 2\}$ for our experiments.

1) *Integer-valued Schedulers:* Recall that schedulers with integer-valued rate outputs can be obtained by setting $\Delta = 1$ in the VIA, starting from any integer scheduler. We compare the performance of the scheduler obtained by our iterative algorithm to the one using TDMA in conjunction with the optimal single user integer schedulers, see [19] for the latter. The average sum-power is plotted as a function of the link parameter α in Figure 13. Observe that the

proposed strategy and TDMA performs equally well when $\alpha = 1$, i.e. when the conditions at both users are identical. But when α moves away from 1, the advantage of using the strategies proposed in this paper is evident.

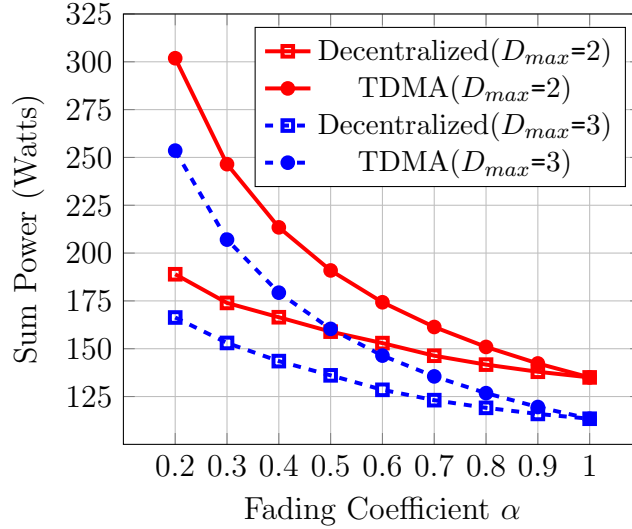


Fig. 13. Integer BiS+Optimal CeN Vs Integer BiS+TDMA, $\alpha_1 = 1, \alpha_2 = \alpha$.

2) *Robust Schedulers with Optimal Power Allocation:* We now show that the performance improvement with respect to TDMA is visible even in rational (non-integer) scheduling setups. In particular, we show that even if one commits to the robust schedulers of [20] at the BiSs, the power efficiency of the allocation in Lemma 10 is superior to the non-integer schedulers based on TDMA. Notice that the robust schedulers are agnostic to the arrival distribution [20]. Figure 14 compares the power expenditure when the link parameter α is varied from 0.2 to 1 for $D_{max} = 2$ as well as $D_{max} = 3$. With reference to Figure 14, a robust time-varying scheduler in conjunction with power allocations of Lemma 10 can be a reasonable choice for distributed scheduling in a MAC with bursty arrivals.

3) *Robust Scheduling Vs Optimal Scheduling:* Let us now design optimal (real-valued) schedulers using the VIA at different step sizes, say $\Delta = 0.5$ and $\Delta = 0.1$, as explained in Section V-B. For $D_{max} = 2$, Figure 15 shows the average sum power of real-valued schedulers at these step sizes, used in conjunction with the optimal power laws of Lemma 10. It can be seen that with a step size 0.5 and less, the proposed scheduler outperforms the robust scheduling framework. Thus, the knowledge of arrival statistics can be put to good use by appropriately factoring these

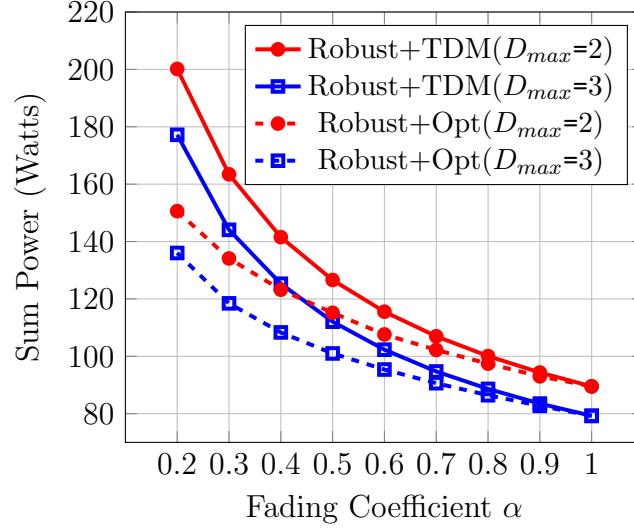


Fig. 14. Robust Schedulers at BiS with Optimal/TDMA Power Allocation, $\alpha_1 = 1, \alpha_2 = \alpha$.

in the dynamic program. Notice also that the performance of a real-valued scheduler may further improve with a reduction in the VIA step size.

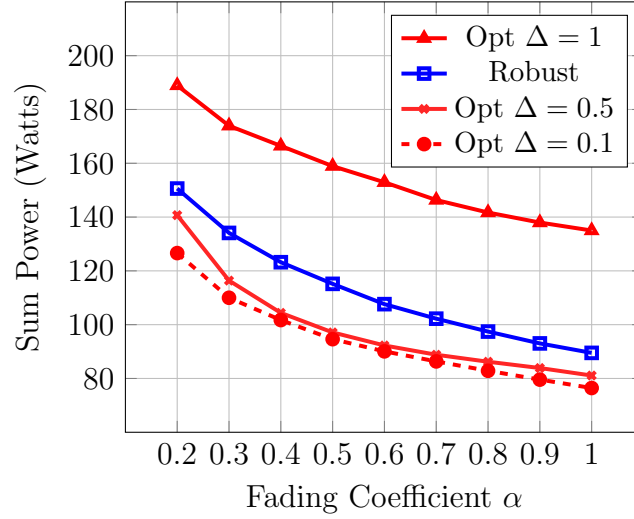


Fig. 15. Performance of schedulers with variable step sizes for $D_{max} = 2$

D. Complexity of Algorithm IterOpt

Notice that Algorithm IterOpt needs to be run only once at the start, before the transmissions begin. Using the arrival statistics from all the users, the algorithm specifies a BiS and a CeN at each transmitter. Since the iterative procedure is to be done only once for a given set of statistics, some level of computational complexity is acceptable, and can be amortized over time. It is reasonable to assume that a single entity computes the communication scheme before any transmission starts, and supplies the relevant rate and power allocation functions to all the terminals. On the other hand, it is also possible for each terminal to separately run Algorithm IterOpt using the available statistics. In the latter case, the complexity mentioned below needs to be scaled by the number of users.

The most computationally intensive part of the Algorithm is Step 2. This involves solving a dynamic program (DP). While closed form solutions are not often available for DPs, approximate solutions are obtained using value iteration or policy iteration [39]. As pointed out in [28], solving MDPs with multiple queues usually leads to a complexity explosion. However, in our algorithm, each terminal solves a separate MDP, and the queues do not interact under a given power allocation scheme. Thus the complexity is linear in the number of users. Given the arrival processes, the number of possible buffer states M and number of possible actions N at each user are determined by the choice of the quantization level Δ . The VIA used in our simulations is of polynomial complexity in both M and N . Thus, very fine quantizations and/or higher values of D_{max} can make the computations intractable. However, there are ways to speed up the MDP computations at the expense of accuracy. In any case, solving the MDP or finding approximate/heuristic solutions thereof seems an unavoidable step in communication schemes minimizing the average transmit sum-power under delay constraints [19].

Step 3 of Algorithm IterOpt is also polynomial in the number of states, as it solves for the stationary distribution of a Markov Chain. For the iterative power allocation scheme, power needs to be assigned once for each and every rate at a terminal, thus the complexity of power allocation is linear in the number of scheduled rates at each terminal. Clearly, we have effectively used the individual arrival statistics in formulating the MDP, whereas the global statistics were used in specifying the power allocation.

VI. CONCLUSION

In this paper, we presented optimal multiuser communication schemes for the transmission of independent bursty traffic over a distributed multiple access channel under a max-delay constraint. An iterative algorithm was proposed to evaluate the minimum average sum-power. While results are given for a two user model, generalizations to N users are possible. The unit slot delay power allocation of Section III is the key to such extensions, as the rest of the results are largely user independent, except for the computational requirements. The many user unit slot delay power allocation for static fading, for example, can be obtained as outlined below.

Let $\alpha_1, \dots, \alpha_L$ be the fading power gains in the descending order. Recall that $b_i(x)$ is the inverse CDF function for the rates arriving at user i . Suppose we scale each probability at user $i \in \{1, \dots, L\}$ by $\frac{\alpha_L}{\alpha_i}$ and place an additional probability mass of value $1 - \frac{\alpha_L}{\alpha_i}$ at zero, to obtain a transformed CDF ψ_i . Let $\Gamma := \{\gamma_0, \dots, \gamma_{|\Gamma|-1}\}$ be the ordered union of the range of $\psi_i, 1 \leq i \leq L$.

Denote $\hat{b}_i(l) := b_i\left((\gamma_l - 1 + \frac{\alpha_L}{\alpha_i})\frac{\alpha_i}{\alpha_L}\right)$ and $P_i(l) := P_i(\hat{b}_i(l))$. Let us now iteratively allocate powers to the rate-tuples $\hat{b}_1(l), \dots, \hat{b}_L(l)$ in such a way that

$$\sum_{i=1}^L \alpha_i P_i(l) = 2^{2 \sum_{i=1}^L \hat{b}_i(l)} - 1.$$

In particular, the allocation

$$\alpha_i P_i(l) = 2^{2(\sum_{j=1}^{i-1} \hat{b}_j(l-1) + \sum_{j=i}^L \hat{b}_j(l))} - 1 - \sum_{j=1}^i \alpha_j P_j(l-1) - \sum_{j=i+1}^L \alpha_j P_j(l), \quad (30)$$

will do the job, starting with an appropriate initial power allocation on the dominant face of the corresponding contra-polymatroid. Notice that this allocation assigns a power to each rate at every user.

Intuitively, each BiS attempts to smoothen the traffic, in such a way that the transmit power is kept steady across slots. In the absence of fading, considerable smoothening can be achieved by even simple techniques such as sending fractions of size $1/D_{max}$ of a packet for D_{max} consecutive slots. The iterative power allocation will now specify the optimal transmit powers. However, more care is required in presence of fading. While Remark 16 helps here, extending the optimal schemes in Section V to both time-varying fading as well as arrivals, under a general max-delay constraint, appears difficult. In presence of fading, even a single user optimal BiS

becomes more complicated to solve. This difficulty can be sidestepped by taking recourse to efficient, but suboptimal, scheduling heuristics at the BiS. We demonstrate the performance of adapting a heuristic policy for the point to point channel from [43], to our MAC model.

Assume a distributed model where each transmitter is aware only of its own arrivals and time varying fading parameters. Let us employ the *Derivative Directed* (DD) online adaptive scheduler proposed by [43] at each BiS. This mimics a water-filling scheme by attempting to maintain the derivative of the power allocation at terminal i . For a given power allocation function $P_i(r, h)$ at user i in slot j , we compute an estimate $D_i[j]$ of the derivative of the power allocation with respect to the rate r , where h is the fading power gain. A rate value $r_i[j]$ is now chosen such that

$$P'_i(r, h_i[j]) = D_i[j].$$

Furthermore, for the buffer state vector $\zeta_i[j]$ at BiS i , the transmission rate $B_i[j]$ in slot j is taken as

$$B_i[j] = \min \left\{ \max \left\{ r_i[j], \max_{1 \leq d \leq D_{max}} \frac{1}{d} \sum_{k=1}^d \zeta_i[k] \right\}, \sum_{d=1}^{D_{max}} \zeta_i[d] \right\}. \quad (31)$$

The derivative estimate is updated in each slot using

$$D_i[j] = \beta D_i[j-1] + (1 - \beta) P'_i(B_i[j], h_i[j]), \quad \text{where } 0 < \beta \leq 1. \quad (32)$$

This scheduling scheme meets the maximum delay constraint. Also, the optimal power allocation for $D_{max} = 1$ from Section IV gives a convenient starting point. Figure 16 below compares the performance of DD online scheduler under equal fraction TDMA and the optimal power allocations, for an example where the arrivals are uniform in $\mathcal{A} = \{0, 1, \dots, 4\}$, and the fading coefficients H_1 and H_2 are chosen uniform in $\{3, 4\}$ and $\{1, 2\}$ respectively.

While we chose a single delay constraint for all the users, the results are expected to hold under different max-delay constraints at the transmitters. Identifying the optimal communication schemes for an average packet-delay constraint is an interesting future-work. Throughput maximization under energy harvesting nodes in a MAC [36] appears to have some dual relations with the average power minimization problem here. Exploring this duality is another future work. Lastly, we have put the knowledge of the arrival statistics to good use in solving the decentralized MAC problem. In principle, one can start with any outage free communication scheme and possibly learn some of statistical parameters from the available resources, using

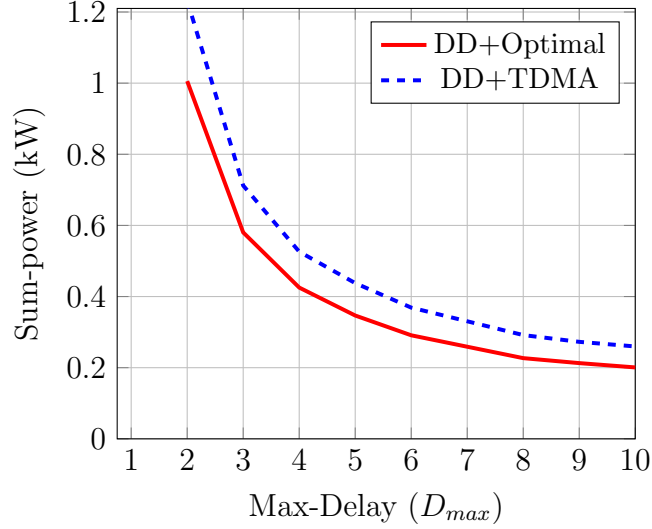


Fig. 16. Dynamic Fading and Arrivals

online learning algorithms [39]. This can then be used to progressively update the schedulers. This scheme, in fact, builds on the proposed solutions here, and will be explored further in future.

While we have stated the results for minimum average sum-power, the CDF transformation technique in [9] can be applied here to evaluate the minimum weighted average sum-power also.

APPENDIX A

PROOF OF LEMMA 3

It is known that for a MDP formulation with bounded costs and finite state-space, there exists a deterministic stationary Markov policy which is average cost optimal [40], [44]. Since we assume bounded arrivals and a maximal delay constraint in our model, the queue-states have bounded entries as well. The essential idea of the proof now is to employ a quantization of the state-space.

Proof: We assumed the transmit power at each terminal to be continuous in the data-rate requirement. Thus for any required transmission rate r , adding a *dummy* rate of $\epsilon > 0$ will cause the required transmit power at that terminal to increase by at most $\delta(\epsilon)$, with $\delta(\epsilon) \rightarrow 0$ as $\epsilon \rightarrow 0$. Note that the utility in (2) is normalized with respect to the number of slots M . Thus, adding

dummy rates of size at most ϵ to each state vector will increase the empirical average power requirement by an amount less than $L\delta(\epsilon)$, which is negligible for small enough ϵ .

Observe that for $\epsilon > 0$, the state-space is discrete with bounded entries. In this case, a deterministic stationary policy solves the average cost MDP formulation [40]. Thus we can limit our search to deterministic stationary policies, where a terminal's scheduling decision is entirely determined by its state-vector, independent of the time of its occurrence. Notice that the discretization makes the state-space and action space finite, implying the existence of the limit in (3). ■

APPENDIX B

PROOF OF THEOREM 14

Proof: Let us first find a lower bound to the average power. Take $P_1(0, 0) = 0$, and $\gamma_{-1} = 0$.

$$\begin{aligned}
& E_{\phi_1, \psi_1} (P_1(B_1, H_1)) + E_{\phi_2, \psi_2} (P_2(B_2, H_2)) \\
&= \sum_{j=1}^{|\mathcal{B}_1|} \sum_k P_1(b_{1j}, h_{1k}) p_{1j} q_{1k} + \sum_{j=1}^{|\mathcal{B}_2|} \sum_k P_2(b_{2j}, h_{2k}) p_{2j} q_{2k} \\
&= \sum_{j=1}^{|\mathcal{B}_1|} \sum_k h_{1k}^2 P_1(b_{1j}, h_{1k}) \frac{p_{1j} q_{1k}}{h_{1k}^2} + \sum_{j=1}^{|\mathcal{B}_2|} \sum_k h_{2k}^2 P_2(b_{2j}, h_{2k}) \frac{p_{2j} q_{2k}}{h_{2k}^2} \\
&= \sum_{j=1}^{|\mathcal{B}_1|} \sum_k h_{1k}^2 P_1(b_{1j}, h_{1k}) \frac{p_{1j} q_{1k}}{h_{1k}^2} + 0 \times P_1(0, 0) [\beta_{|\mathcal{B}_2||\mathcal{H}_2|} - \alpha_{|\mathcal{B}_1||\mathcal{H}_1|}] \\
&\quad + \sum_{l=0}^{|\Gamma|-1} h_2^2(\gamma_l) P_2(b_2(\gamma_l), h_2(\gamma_l)) [\gamma_l - \gamma_{l-1}] \\
&= \sum_{l=0}^{|\Gamma|-1} h_1^2(\gamma_l) P_1(b_1(\gamma_l), h_1(\gamma_l)) [\gamma_l - \gamma_{l-1}] + \sum_{l=0}^{|\Gamma|-1} h_2^2(\gamma_l) P_2(b_2(\gamma_l), h_2(\gamma_l)) [\gamma_l - \gamma_{l-1}] \\
&= \sum_{l=0}^{|\Gamma|-1} [h_1^2(\gamma_l) P_1(b_1(\gamma_l), h_1(\gamma_l)) + h_2^2(\gamma_l) P_2(b_2(\gamma_l), h_2(\gamma_l))] [\gamma_l - \gamma_{l-1}]. \tag{33}
\end{aligned}$$

Now an outage-free power allocation should satisfy

$$h_1^2(\gamma_l) P_1(b_1(\gamma_l), h_1(\gamma_l)) + h_2^2(\gamma_l) P_2(b_2(\gamma_l), h_2(\gamma_l)) \geq 2^{2(b_1(\gamma_l) + b_2(\gamma_l))} - 1. \tag{34}$$

Thus

$$P_{avg}^{min}(1) \geq \sum_{l=0}^{|\Gamma|-1} [2^{2(b_1(\gamma_l)+b_2(\gamma_l))} - 1] [\gamma_l - \gamma_{l-1}]. \quad (35)$$

But the RHS is indeed achieved by the power allocations in (22) – (25). More specifically, (23) ensures equality in (34) for every $\gamma_l \in \Gamma$. ■

APPENDIX C

PROOF OF LEMMA 15

The essential ingredient for the proof is given in the lemma below.

Lemma 20. *Let b_1, b'_1, b_2, b'_2 be rates such that $b'_1 \geq b_1$ and $b'_2 \geq b_2$, and let h_1, h'_1, h_2, h'_2 be arbitrary fading values. Let the power allocation functions $P_1(\cdot)$ and $P_2(\cdot)$ satisfy $h_2^2 P_2(b_2) + h_1^2 P_1(b_1) \geq 2^{2(b_1+b_2)} - 1$ and $h_2'^2 P_2(b'_2) + h_1'^2 P_1(b'_1) \geq 2^{2(b'_1+b'_2)} - 1$. If in addition $h_2'^2 P_2(b'_2) + h_1^2 P_1(b_1) = 2^{2(b_1+b'_2)} - 1$, then*

$$h_2^2 P_2(b_2) + h_1'^2 P_1(b'_1) \geq 2^{2(b'_1+b_2)} - 1.$$

Proof: Observe that

$$\begin{aligned} h_2^2 P_2(b_2) + h_1'^2 P_1(b'_1) &= h_2^2 P_2(b_2) + h_1^2 P_1(b_1) + h_2'^2 P_2(b'_2) + h_1'^2 P_1(b'_1) - (h_2'^2 P_2(b'_2) + h_1^2 P_1(b_1)) \\ &\geq 2^{2(b_1+b_2)} + 2^{2(b'_1+b'_2)} - 2^{2(b_1+b'_2)} - 1. \end{aligned} \quad (36)$$

Note that $b_1 + b_2 \leq b'_1 + b_2 \leq b'_1 + b'_2$ and $b_1 + b_2 \leq b_1 + b'_2 \leq b'_1 + b'_2$. Thus,

$$2^{2(b'_1+b'_2)} + 2^{2(b_1+b_2)} \geq 2^{2(b_1+b'_2)} + 2^{2(b'_1+b_2)},$$

by the convexity of 2^{2x} and Jensen's inequality. The lemma now follows from (36). ■

Let us now prove Lemma 15.

Proof: Consider any rate-channel pair (b_{1j}, h_{1k}) and (b_{2m}, h_{2n}) of user 1 and 2 respectively.

We will show that

$$h_{1k}^2 P_1(b_{1j}, h_{1k}) + h_{2n}^2 P_2(b_{2m}, h_{2n}) \geq 2^{2(b_{1j}+b_{2m})} - 1. \quad (37)$$

From the definition of γ_l , it follows that

$$h_{1k}^2 P_1(b_{1j}, h_{1k}) = h_1^2(\gamma_{l_1}) P_1(b_1(\gamma_{l_1}), h_1(\gamma_{l_1}))$$

$$h_{2n}^2 P_1(b_{1m}, h_{1n}) = h_2^2(\gamma_{l_2}) P_2(b_2(\gamma_{l_2}), h_2(\gamma_{l_2})), \quad (38)$$

for some $0 \leq l_1 \leq |\Gamma| - 1$, $0 \leq l_2 \leq |\Gamma| - 1$. So we need to prove that

$$h_1^2(\gamma_{l_1}) P_1(b_1(\gamma_{l_1}), h_1(\gamma_{l_1})) + h_2^2(\gamma_{l_2}) P_2(b_2(\gamma_{l_2}), h_2(\gamma_{l_2})) \geq 2^{2(b_1(\gamma_{l_1}) + b_2(\gamma_{l_2}))} - 1. \quad (39)$$

If $l_1 = l_2$, then (39) follows trivially from (23). Assume without loss of generality that $l_1 > l_2$.

The opposite case can be handled in a similar fashion. Suppose it holds that

$$h_1^2(\gamma_{l_1-1}) P_1(b_1(\gamma_{l_1-1}), h_1(\gamma_{l_1-1})) + h_2^2(\gamma_{l_2}) P_2(b_2(\gamma_{l_2}), h_2(\gamma_{l_2})) \geq 2^{2(b_1(\gamma_{l_1-1}) + b_2(\gamma_{l_2}))} - 1. \quad (40)$$

Using this, along with (22) and (23) appropriately in Lemma 20, it follows that

$$h_1^2(\gamma_{l_1}) P_1(b_1(\gamma_{l_1}), h_1(\gamma_{l_1})) + h_2^2(\gamma_{l_2}) P_2(b_2(\gamma_{l_2}), h_2(\gamma_{l_2})) \geq 2^{2(b_1(\gamma_{l_1}) + b_2(\gamma_{l_2}))} - 1. \quad (41)$$

Thus by induction on l_1 , (39) holds for any $l_1 > l_2$. We next show that for $i = 1, 2$,

$$h_i^2(\gamma_l) P_i(b_i(\gamma_l), h_i(\gamma_l)) \geq 2^{2b_i(\gamma_l)} - 1. \quad (42)$$

We prove the case for $i = 1$ by induction (the case of $i = 2$ is similar). The initial step in the induction is given by (25). Let

$$h_1^2(\gamma_{l-1}) P_1(b_1(\gamma_{l-1}), h_1(\gamma_{l-1})) \geq 2^{2b_1(\gamma_{l-1})} - 1. \quad (43)$$

Then,

$$\begin{aligned} h_1^2(\gamma_l) P_1(b_1(\gamma_l), h_1(\gamma_l)) &= (h_1^2(\gamma_l) P_1(b_1(\gamma_l), h_1(\gamma_l)) + h_2^2(\gamma_l) P_2(b_2(\gamma_l), h_2(\gamma_l))) \\ &\quad - (h_1^2(\gamma_{l-1}) P_1(b_1(\gamma_{l-1}), h_1(\gamma_{l-1})) + h_2^2(\gamma_l) P_2(b_2(\gamma_l), h_2(\gamma_l))) \\ &\quad + h_1^2(\gamma_{l-1}) P_1(b_1(\gamma_{l-1}), h_1(\gamma_{l-1})) \\ &\geq 2^{2(b_1(\gamma_l) + b_2(\gamma_l))} - 2^{2(b_1(\gamma_{l-1}) + b_2(\gamma_l))} + 2^{2b_1(\gamma_{l-1})} - 1 \end{aligned} \quad (44)$$

$$\begin{aligned} &= 2^{2(b_1(\gamma_l) + b_2(\gamma_l))} - 2^{2b_1(\gamma_{l-1})} (2^{2b_2(\gamma_l)} - 1) - 1 \\ &\geq 2^{2(b_1(\gamma_l) + b_2(\gamma_l))} - 2^{2b_1(\gamma_l)} (2^{2b_2(\gamma_l)} - 1) - 1 \\ &= 2^{2b_1(\gamma_l)} - 1. \end{aligned} \quad (45)$$

Here (44) follows from (22), (23) and (43). Notice that (45) follows from the fact that $b_1(\gamma_{l-1}) \leq b_1(\gamma_l)$. This proves the result. ■

APPENDIX D

PROOF OF LEMMA 12

The proof is similar to that in Lemma 7, we present it here for completeness.

Proof: Consider three required packet-rates b'_1, b_1, b''_1 at user 1 in the ascending order. W.l.o.g, take $b_1 = \lambda b'_1 + (1 - \lambda)b''_1$ for some $\lambda \in (0, 1)$. To prove the lemma, we will show that

$$P_1(b_1) \leq \lambda P_1(b'_1) + (1 - \lambda)P_1(b''_1).$$

By the power allocation in Lemma 10, we know that for some $b_2 \in \mathcal{B}_2$, the rate-pair (b_1, b_2) was assigned power from the dominant face of a corresponding contra-polymatroid, i.e.

$$\alpha_1 P_1(b_1) + \alpha_2 P_2(b_2) = 2^{2(b_1+b_2)} - 1.$$

We also know that for $\tilde{b} \in \{b'_1, b''_1\}$

$$\alpha_1 P_1(\tilde{b}) + \alpha_2 P_2(b_2) \geq 2^{2(\tilde{b}+b_2)} - 1. \quad (46)$$

Taking a λ -linear combination, and using convexity,

$$\begin{aligned} \alpha_1(\lambda P_1(b'_1) + (1 - \lambda)P_1(b''_1)) + \alpha_2 P_2(b_2) &\geq \lambda 2^{2(b'_1+b_2)} + (1 - \lambda)2^{2(b''_1+b_2)} - 1 \\ &\geq 2^{2(\lambda b'_1 + (1-\lambda)b''_1 + b_2)} - 1 \\ &= 2^{2(b_1+b_2)} - 1 \\ &= \alpha_1 P_1(b_1) + \alpha_2 P_2(b_2). \end{aligned} \quad (47)$$

■

APPENDIX E

PROOF OF PROPOSITION 19

For the BiSs S_1 and S_2 , let $P_{s_1}(\cdot)$ and $P_{s_2}(\cdot)$ be the respective optimal power allocations obtained by Lemma 10. By a slight abuse of notation, let us denote by $P_{avg}(S_1, S_2)$ the average transmit sum-power achieved by employing (S_1, P_{s_1}) and (S_2, P_{s_2}) respectively at the two transmitters. We first show that the average sum-power can be optimized by alternating the minimization of $P_{avg}(S_1, S_2)$ between S_1 and S_2 . On the other hand, though Algorithm IterOpt alternates between (S_1, S_2) and (P_1, P_2) , it still manages to find the same minimum. We start with the following lemma.

Lemma 21. $P_{avg}(S_1, S_2)$ is strictly convex in S_1 for a given S_2 .

Proof: Consider two possible BiS schemes S_a and S_b for user 1, and let the second user employ the BiS S_2 . Let (P_{1a}, P_{2a}) and (P_{1b}, P_{2b}) denote the optimal power allocation schemes under the pair of schedulers (S_a, S_2) and (S_b, S_2) respectively. For $j \in \{a, b\}$, the average sum-power required at user l is denoted as P_{lj}^{sum} , $l \in \{1, 2\}$. Now, Lemma 7 guarantees that a λ -linear combination of (S_a, S_2) and (S_b, S_2) will be an outage free scheme. The average sum-power required for such a policy is $\lambda P_{1a}^{sum} + (1 - \lambda)P_{1b}^{sum} + \lambda P_{2a}^{sum} + (1 - \lambda)P_{2b}^{sum}$. It turns out that we can strictly improve this, when S_a and S_b are not identical. Assume that there exists a rate b_a (b_b) scheduled in S_a (S_b) such that $b_\lambda = \lambda b_a + (1 - \lambda)b_b$ is scheduled at BiS S_1 , and $b_a \neq b_b$. We now show that the power allocation $P_{1\lambda}$ (linear combination of P_{1a} and P_{1b}) at user 1 is strictly sub-optimal. In particular, $P_{1\lambda}(\cdot)$ fails to allocate power for the rate b_λ from the dominant face of any feasible contra-polymatroid. Thus, the power for b_λ can be decreased without violating any other constraint or allocations. To see this, for any b_2 scheduled at user 2, we have

$$\alpha_1 P_{1\lambda}(b_\lambda) + \alpha_2 P_{2\lambda}(b_2) \geq \lambda 2^{2(b_a+b_2)} + (1 - \lambda)2^{2(b_b+b_2)} - 1 \quad (48)$$

$$> 2^{2(\lambda b_a + (1-\lambda)b_b + b_2)} - 1. \quad (49)$$

The last inequality results from the strict convexity of 2^x , $x \geq 0$. Thus, we can decrease $P_{1\lambda}(b_\lambda)$ by a sufficiently small positive amount, and still guarantee the outage free nature of the scheme. ■

Let us denote the minimal value of $P_{avg}(S_1, S_2)$ over (S_1, S_2) as P_{AM}^* . Consider an alternating minimization algorithm for minimizing $P_{avg}(S_1, S_2)$ over all feasible distributed stationary schedulers. Lemma 7 and Lemma 21 ensures that alternating the iterations between S_1 and S_2 will converge to the optimal value P_{AM}^* . This follows from the well known theory of alternating minimization [42], [45]. However, such an alternation among variables is not straight forward in our framework. In particular, the optimal power allocation in Lemma 10 is jointly evaluated using the marginal CDFs at the output of both the schedulers S_1 and S_2 . While Algorithm IterOpt circumvented this issue by alternating over the variables (S_1, S_2) and (P_1, P_2) , fortunately, its terminal average power P_{HALT}^* still yields the correct minimum, i.e.

$$P_{HALT}^* = P_{AM}^* = P_{avg}^{min}(D_{max}).$$

To see this, let $C(P_1, P_2)$ denote the average sum-power for power policies P_1 and P_2 at the respective users. The associated schedulers will be clear from the context. Assume that Algorithm IterOpt terminates by converging to the BiS-CeN pairs (S_1^*, P_1^*) and (S_2^*, P_2^*) for users 1 and 2 respectively. Observe that S_2^* is an optimal rate scheduler for the power control law P_2^* (see Claim 18). In order to show that (S_1^*, P_1^*) and (S_2^*, P_2^*) are optimal, let us now perform an alternate minimization between (S_1, P_1) and (S_2, P_2) . For contradiction, assume that (S_1^*, S_2^*) is not the optimal choice. W.l.o.g, suppose we start with (S_1^*, P_1^*) at the first user, and obtain another pair (S_2', P_2') such that $P_2^* \neq P_2'$ and

$$C(P_1^*, P_2^*) > C(P_1^*, P_2'). \quad (50)$$

The inequality (50) suggests that the point (S_2^*, P_2^*) obtained via Algorithm IterOpt was not the true optimum. Using P_2^* and P_2' , let us construct another power function $P_2^o = \min(P_2^*, P_2')$. Clearly,

$$C(P_1^*, P_2') > C(P_1^*, P_2^o).$$

Notice that (S_2^*, P_2^o) is also a feasible scheduler-power pair for user 2, and does not cause outage with any rate of user 1. The average sum-power under the new power allocation (P_1^*, P_2^o) is strictly lower than that of either (P_1^*, P_2^*) or (P_1^*, P_2') . However P_2^* is an optimal power allocation function for S_2^* . Hence the power-rate characteristics of P_2^* and P_2' must be identical. Once P_2^* is fixed, S_2^* is indeed an optimal scheduler by Claim 18. Thus (S_1^*, S_2^*) is indeed the stationary point of an alternating minimization algorithm [42], and in lieu of Lemma 21 and Lemma 7, it achieves the optimal value.

APPENDIX F

CONTINUOUS VALUED PACKET ARRIVALS WITH UNIT SLOT DELAY CONSTRAINT

Consider packet arrivals with continuous valued rate requirements under a unit delay constraint. Let the rate requirement be B_i with respective CDFs $\phi_i(\cdot)$ for user i . For notational convenience, assume the fading coefficients of user 1 and user 2 to be 1 and $\sqrt{\alpha}$ respectively, with $\alpha \leq 1$. Define $\tilde{\phi}_1(x) := 1 - \alpha + \alpha\phi_1(x)$ and $\tilde{\phi}_2(x) := \phi_2(x)$. Let $\tilde{b}_i(y) := \tilde{\phi}_i^{-1}(y)$, $i = 1, 2$, as given in (8). The following power allocation minimizes the average sum power for $D_{max} = 1$.

Theorem 22. *The power allocations*

$$P_1(\tilde{b}_1(x)) = P_1(\tilde{b}_1(0)) + 2 \int_{\tilde{b}_1(0)}^{\tilde{b}_1(x)} 2^{2(y+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(y)))} dy, \quad 0 \leq x \leq 1 \quad (51)$$

$$P_2(\tilde{b}_2(x)) = P_2(\tilde{b}_2(0)) + \frac{2}{\alpha} \int_{\tilde{b}_2(0)}^{\tilde{b}_2(x)} 2^{2(y+\tilde{\phi}_1^{-1}(\tilde{\phi}_2(y)))} dy, \quad 0 \leq x \leq 1 \quad (52)$$

for any $P_1(\tilde{b}_1(0)), P_2(\tilde{b}_2(0))$ such that

$$P_1(\tilde{b}_1(0)) \geq 2^{2\tilde{b}_1(0)} - 1 \quad (53)$$

$$\alpha P_2(\tilde{b}_2(0)) \geq 2^{2\tilde{b}_2(0)} - 1 \quad (54)$$

$$P_1(\tilde{b}_1(0)) + \alpha P_2(\tilde{b}_2(0)) = 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} - 1 \quad (55)$$

achieves $P_{avg}^{min}(1)$.

Proof: First, we prove a lower bound on the sum-power. Using the steps in (10) – (12), we get

$$\mathbb{E}P_1(B_1) + \mathbb{E}P_2(B_2) \geq \int_0^{1-\alpha} \frac{2^{2b_2(x)} - 1}{\alpha} dx + \int_{1-\alpha}^1 \frac{2^{2(b_1(\frac{x-1+\alpha}{\alpha})+b_2(x))} - 1}{\alpha}.$$

Since $\tilde{b}_1(x) = b_1(\frac{x-1+\alpha}{\alpha})$ for $1-\alpha \leq x \leq 1$, we have

$$P_{avg}^{min}(1) \geq \int_0^{1-\alpha} \frac{2^{2b_2(x)} - 1}{\alpha} dx + \int_{1-\alpha}^1 \frac{2^{2(\tilde{b}_1(x)+b_2(x))} - 1}{\alpha}. \quad (56)$$

Next, we show that the power allocation given in (51) and (52) can achieve the lower bound. For $y \leq 1-\alpha$, we have $\tilde{\phi}_1^{-1}(\tilde{\phi}_2(y)) = 0$ and $\tilde{b}_1(y) = 0$. Thus $P_1(\tilde{b}_1(y)) = 0$, and

$$P_2(b_2(y)) = \frac{2^{2b_2(y)} - 1}{\alpha} \quad (57)$$

For $1-\alpha \leq x \leq 1$,

$$\begin{aligned} & P_1(\tilde{b}_1(x)) + \alpha P_2(\tilde{b}_2(x)) \\ &= P_1(\tilde{b}_1(0)) + 2 \int_{\tilde{b}_1(0)}^{\tilde{b}_1(x)} 2^{2(y+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(y)))} dy + \alpha P_2(\tilde{b}_2(0)) + 2 \int_{\tilde{b}_2(0)}^{\tilde{b}_2(x)} 2^{2(y+\tilde{\phi}_1^{-1}(\tilde{\phi}_2(y)))} dy. \end{aligned}$$

Substituting $\tilde{\phi}_2^{-1}(\tilde{\phi}_1(y)) = z$, we get

$$P_1(\tilde{b}_1(x)) + \alpha P_2(\tilde{b}_2(x)) = P_1(\tilde{b}_1(0)) + \alpha P_2(\tilde{b}_2(0)) + 2 \int_{\tilde{b}_2(0)}^{\tilde{b}_2(x)} 2^{2(\tilde{\phi}_1^{-1}(\tilde{\phi}_2(z))+z)} d(\tilde{\phi}_1^{-1}(\tilde{\phi}_2(z)))$$

$$\begin{aligned}
& + 2 \int_{\tilde{b}_2(0)}^{\tilde{b}_2(x)} 2^{2(y+\tilde{\phi}_1^{-1}(\tilde{\phi}_2(y)))} dy \\
& = P_1(\tilde{b}_1(0)) + \alpha P_2(\tilde{b}_2(0)) + 2 \int_{\tilde{b}_2(0)}^{\tilde{b}_2(x)} 2^{2(\tilde{\phi}_1^{-1}(\tilde{\phi}_2(z))+z)} d(\tilde{\phi}_1^{-1}(\tilde{\phi}_2(z)) + z) \\
& = P_1(\tilde{b}_1(0)) + \alpha P_2(\tilde{b}_2(0)) + 2^{2(\tilde{b}_1(x)+\tilde{b}_2(x))} - 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} \tag{58}
\end{aligned}$$

$$= 2^{2(\tilde{b}_1(x)+\tilde{b}_2(x))} - 1. \tag{59}$$

Now, (57) and (59) imply that we have equality in (56). Note that from the above discussion, the sum power can be written as

$$\mathbb{E}P_1(B_1) + \mathbb{E}P_2(B_2) = \int_0^1 \left(2^{2(\tilde{b}_1(x)+\tilde{b}_2(x))} - 1 \right) dx. \tag{60}$$

Next we show that the power allocations in Theorem 22 are outage free. Substituting the lower limit of integration in $\tilde{\phi}_2^{-1}(\tilde{\phi}_1(y))$ in (51) and (52), we get

$$\begin{aligned}
P_1(b_1) &= P_1(\tilde{b}_1(0)) + 2 \int_{\tilde{b}_1(0)}^{b_1} 2^{2(y+\tilde{b}_2(0))} dy \\
&\geq 2^{2\tilde{b}_1(0)} - 1 + 2 \int_{\tilde{b}_1(0)}^{b_1} 2^{2(y+\tilde{b}_2(0))} dy \\
&= 2^{2\tilde{b}_1(0)} - 1 + 2^{2\tilde{b}_2(0)} (2^{2b_1} - 2^{2\tilde{b}_1(0)}) \\
&\geq 2^{2b_1} - 1.
\end{aligned}$$

Similarly, we have $P_2(b_2) \geq \frac{1}{\alpha} (2^{2b_2} - 1)$. Furthermore

$$\begin{aligned}
& P_1(b_1) + \alpha P_2(b_2) \\
&= P_1(\tilde{b}_1(0)) + \alpha P_2(\tilde{b}_2(0)) + 2 \left[\int_{\tilde{b}_1(0)}^{b_1} 2^{2(y+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(y)))} dy + \int_{\tilde{b}_2(0)}^{b_2} 2^{2(y+\tilde{\phi}_1^{-1}(\tilde{\phi}_2(y)))} dy \right].
\end{aligned}$$

Substituting $\tilde{\phi}_1^{-1}(\tilde{\phi}_2(y)) = z$ in the second integral above, we get

$$\begin{aligned}
P_1(b_1) + \alpha P_2(b_2) &= 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} - 1 + 2 \int_{\tilde{b}_1(0)}^{b_1} 2^{2(y+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(y)))} dy \\
&\quad + 2 \int_{\tilde{b}_1(0)}^{\tilde{\phi}_1^{-1}(\tilde{\phi}_2(b_2))} 2^{2(z+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z)))} d(\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z))).
\end{aligned}$$

Now, suppose $\tilde{\phi}_1^{-1}(\tilde{\phi}_2(b_2)) \geq b_1$. Then

$$P_1(b_1) + \alpha P_2(b_2) = 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} - 1 + 2 \int_{\tilde{b}_1(0)}^{b_1} 2^{2(y+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(y)))} d(y + \tilde{\phi}_2^{-1}(\tilde{\phi}_1(y)))$$

$$\begin{aligned}
& + 2 \int_{b_1}^{\tilde{\phi}_1^{-1}(\tilde{\phi}_2(b_2))} 2^{2(z+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z)))} d(\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z))) \\
& = 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} - 1 + 2^{2(b_1+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(b_1)))} - 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} \\
& \quad + 2 \int_{b_1}^{\tilde{\phi}_1^{-1}(\tilde{\phi}_2(b_2))} 2^{2(z+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z)))} d(\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z))) \tag{61}
\end{aligned}$$

$$\begin{aligned}
& \geq 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} - 1 + 2^{2(b_1+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(b_1)))} - 2^{2(\tilde{b}_1(0)+\tilde{b}_2(0))} \\
& \quad + 2 \int_{b_1}^{\tilde{\phi}_1^{-1}(\tilde{\phi}_2(b_2))} 2^{2(b_1+\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z)))} d(\tilde{\phi}_2^{-1}(\tilde{\phi}_1(z))) \tag{62}
\end{aligned}$$

$$\geq 2^{2(b_1+b_2)} - 1. \tag{63}$$

The inequality in (62) was obtained by substituting the lower bound of z in (61). The other case when $\tilde{\phi}_1^{-1}(\tilde{\phi}_2(b_2)) < b_1$ can be handled in a similar fashion. Thus the given power allocations are outage free, which proves the theorem. ■

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